MODELLING DOPAMINE AND GLUTAMATE SIGNAL INTEGRATION INFLUENCE ON NEUROADAPTATION

Lu Li
St John’s College

A Dissertation submitted to the University of Cambridge for the degree of Doctor of Philosophy

European Molecular Biology Laboratory
European Bioinformatics Institute
Wellcome Trust Genome Campus
Hinxton, Cambridge, CB10 1SD
United Kingdom

luli@ebi.ac.uk

9th August 2010
Lu Li: *Modelling Dopamine and Glutamate Signal Integration Influence on Neuroadaptation*, A Dissertation submitted to the University of Cambridge for the degree of Doctor of Philosophy, © 9th August 2010

Supervisor:
Nicolas Le Novère

Thesis Advisory Committee:
Stephen Eglen, Cambridge University
Wolfgang Huber, EMBL Heidelberg
Liliana Minichiello, EMBL Monterotondo

Cambridge, 9th August 2010
To my parents
DECLARATION

This dissertation is my own work and includes nothing which is the outcome of work done in collaboration except as specified in the text. It is not substantially the same as any I have submitted for a degree, diploma or other qualification at any other university; and no part has already been, or is currently being submitted for any degree, diploma or other qualification.

This dissertation does not exceed the specified length limit of 300 single-sided pages of double-spaced text as defined by The Biology Degree Committee.

Cambridge, 9th August 2010

Lu Li
Mechanisms underlying reward processes and drug addiction include the neuroadaptation of striatal cells. Integration of cortico-striatal glutamate input and mesencephalic dopamine signalling has been shown to be important. In this thesis, I investigate this transmission cross-talk by studying quantitative models of key molecular processes involved in short-term synaptic plasticity and long-term cellular remodelling.

Using a detailed quantitative computational model of calmodulin function, I investigate the effect of calcium signals by varying spike frequency, spike amplitude, and signal duration. I show that the effective activation of calmodulin depends on calcium spike frequencies rather than the total amount of calcium ions. Furthermore, high frequencies of calcium inputs shift the balance of relative activation from calcineurin to calcium/calmodulin-dependent protein kinase II (CaMKII), inducing long-term potentiation. Dopamine-mediated inhibition of protein phosphatase 1 (PP1) extends the activation of CaMKII by high calcium spiking frequencies, therefore modulating short-term synaptic plasticity.

I also investigate how dopamine and glutamate signals converge on the activity of extracellular signal regulated kinase (ERK), a crucial protein regulating structural changes such as dendritic remodelling and density of synaptic connections. Comparison of alternative models, where either calcineurin or PP1 directly stimulates striatal-enriched tyrosine phosphatase (STEP), with existing experimental data, provides evidence for a direct stimulation of STEP by PP1. However, my experiments conducted on primary striatal neuron cultures after one week or two weeks show a switch of glutamate influence on ERK activity, from activation to inhibition. Detailed computational models provide a tentative explanation why dopamine regulated PP1 activ-
ity displays opposite effects on ERK activity according to different neuronal differentiation and, in particular, the type of N-methyl-D-aspartate (NMDA) receptors expressed.

This thesis sheds light on some biochemical cascades involved in striatal neuroadaptation, and provides accurate models that can be reused and extended to study other aspects of this neuroadaptation.
ACKNOWLEDGEMENTS

First of all, I would like to thank my supervisor Nicolas Le Novère for his support and supervision. I am very grateful for having worked in the exciting field of computational modelling, as well as being involved in wet lab experiments.

I would like to express my gratitude to the members of my thesis advisory committee: Stephen Eglen, Wolfgang Huber and Liliana Minichello, for providing invaluable comments and suggestions.

Many thanks to Denis Hervé and Jean-Antoine Girault for their patience and encouragement while I was in the lab in Paris. I would also like to give special thanks to Miriam Sanchez Matamala and Emmanuelle Jordi, who spared their precious time to teach me how to successfully perform some crucial experiments and made my stay in Paris so enjoyable.

I have a special thought for everyone in the Compneur group: thank you for making me feel involved in a big family.

I want to say Xie Xie to my parents, Camille Laibe, Chen Li, and Penny Coggill for love and understanding.

Finally, I want to thank my supervisor again, without him, it would have been impossible to finish this thesis on time.
CONTENTS

List of figures   x
List of tables   xiv
1 Introduction  1
   1.1 Synaptic basis of neuroadaptation  1
      1.1.1 Reward and neuroadaptation  1
      1.1.2 Synaptic plasticity  3
      1.1.3 Signalling pathways responding to dopamine inputs  10
   1.2 Modelling signalling pathways in neurons  13
      1.2.1 Deterministic simulations of continuous variables  15
      1.2.2 Population-based stochastic simulations  16
      1.2.3 Particle-based stochastic approach  17
2 Short-term synaptic plasticity  18
   2.1 Introduction  18
   2.2 Modelling methods  23
      2.2.1 Modelling and simulation software  23
      2.2.2 Reaction and parameters  23
      2.2.3 Pathway activation  24
   2.3 Calcium controls the relative activation  25
      2.3.1 Model structure  25
      2.3.2 Calcium spikes and simulation design  35
      2.3.3 Parameter definition: the activated area  37
      2.3.4 Frequency-regulated activation of calmodulin  38
      2.3.5 Frequency-regulated calmodulin binding  39
      2.3.6 Frequency-modulated activation of calcineurin and CaMKII  41
      2.3.7 CaMKII autophosphorylation, a calcium input frequency decoder  44
      2.3.8 Total amount of calcium ions  46
      2.3.9 Calmodulin availability  50
      2.3.10 PP1 inhibition by DARPP-32  52
   2.4 Discussion  54
3 Structural plasticity  57
3.1 Introduction 57
3.2 Experimental methods 60
  3.2.1 Cell culture and glutamate stimulation 61
  3.2.2 Western blotting 62
  3.2.3 Immunofluorescence of striatal neurons in culture 62
3.3 Phosphatase regulating STEP activity 62
3.4 Experimental investigation of PP1 inhibitors on ERK activation 89
3.5 Dual influence of PP1 on ERK activity 99
3.6 Discussion 111

4 Discussion 115
  4.1 Synaptic plasticity in NAc 115
  4.2 Structural plasticity in NAc 117
  4.3 Experience-dependent adaptation 119
  4.4 Simulation approach and parameter estimation 121

Bibliography 125
List of abbreviations 164
Publications 167
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Dopamine-glutamate interactions in the nucleus accumbens and the cortico-striato-thalamic loop</td>
<td>2</td>
</tr>
<tr>
<td>Figure 2</td>
<td>NMDA receptor-dependent LTP and LTD</td>
<td>4</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Actin-based spine morphology changes and stabilisation</td>
<td>7</td>
</tr>
<tr>
<td>Figure 4</td>
<td>The involvement of gene expression and protein synthesis in spine morphogenesis</td>
<td>9</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Regulation of DARPP-32 phosphorylation</td>
<td>12</td>
</tr>
<tr>
<td>Figure 6</td>
<td>The model scheme of calcium regulated pathway</td>
<td>22</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Allosteric model of calmodulin by Stefan et al.</td>
<td>27</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Calculation the rate of CaMKII autophosphorylation</td>
<td>29</td>
</tr>
<tr>
<td>Figure 9</td>
<td>The fitted polynomial function for the rate of CaMKII autophosphorylation</td>
<td>30</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Intracellular free calcium concentration increase induced by a single calcium input</td>
<td>36</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Intracellular free calcium concentration increase induced by a train of calcium inputs.</td>
<td>37</td>
</tr>
<tr>
<td>Figure 12</td>
<td>The definition of ‘activated area’</td>
<td>38</td>
</tr>
<tr>
<td>Figure 13</td>
<td>Effects of calcium input frequencies on activation of calmodulin</td>
<td>39</td>
</tr>
<tr>
<td>Figure 14</td>
<td>The ratio of calmodulin bound to calcineurin versus that bound to CaMKII</td>
<td>40</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Effects of a low-frequency calcium signal on activation of calcineurin and CaMKII</td>
<td>42</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Effects of a high-frequency calcium signal on activation of calcineurin and CaMKII</td>
<td>42</td>
</tr>
<tr>
<td>Figure 17</td>
<td>Comparison of calcineurin and CaMKII activation induced by calcium inputs at different frequencies</td>
<td>43</td>
</tr>
</tbody>
</table>
Figure 18  Effect of CaMKII autophosphorylation on the relative activation of calcineurin and CaMKII  
45
Figure 19  Effect of calcium input number on the relative activation of calcineurin and CaMKII  
47
Figure 20  Intracellular free calcium concentration increase induced by a large calcium input  
48
Figure 21  Effect of calcium input size on the relative activation of calcineurin and CaMKII  
49
Figure 22  Effect of calmodulin concentration on the relative activation of calcineurin and CaMKII  
51
Figure 23  Effect of increased PP1 inhibition on the relative activation of calcineurin and CaMKII  
53
Figure 24  Computational models of the regulation of STEP activity  
64
Figure 25  A train of calcium spikes at 50 Hz  
78
Figure 26  A single spike of cAMP increase  
79
Figure 27  Stimulation by calcium and cAMP  
79
Figure 28  Effects of a train of Ca\(^{2+}\) spikes in the model where PP1 activates STEP  
80
Figure 29  Effects of a single cAMP spike in the model where PP1 activates STEP  
81
Figure 30  Effects of a single cAMP pulse and a train of Ca\(^{2+}\) spikes in the model where PP1 activates 
STEP  
82
Figure 31  Effects of a single cAMP pulse and a train of 
Ca\(^{2+}\) spikes in the model where PP1 activates 
STEP and DARPP-32 knock out  
83
Figure 32  ERK activity after calcineurin or PP1 inhibition followed by a train of Ca\(^{2+}\) spikes, in the model 
where PP1 activates STEP  
84
Figure 33  Effects of a train of Ca\(^{2+}\) spikes in the model where calcineurin activates STEP  
85
Figure 34  Effects of a single cAMP spike in the model where calcineurin activates STEP  
86
Figure 35  Effects of a single cAMP pulse and a train of 
Ca\(^{2+}\) spikes in the model where calcineurin acti-

 xi
Figure 36  The ERK activity after calcineurin or PP1 inhibition followed by a train of Ca^{2+} spikes, in the model where calcineurin activates STEP  88

Figure 37  ERK2 activity in response to PP1 inhibition and glutamate stimulation in 7-day neuronal cultures  90

Figure 38  Western blot image of ERK1/2 activity in response to PP1 inhibition and glutamate stimulation in 7-day neuronal cultures  91

Figure 39  ERK2 activity in response to PP1 inhibition and glutamate stimulation in 14-day cultured neurons  92

Figure 40  Western blot image of ERK1/2 activity in response to PP1 inhibition and glutamate stimulation in 14-day neuronal cultures  93

Figure 41  ERK2 activity in response to PP2A inhibition and glutamate stimulation in 14-day neuronal cultures  94

Figure 42  Immunofluorescence image of ERK1/2 activation at basal level in 14-day neuronal cultures  95

Figure 43  Immunofluorescence image on ERK1/2 activation after 10 minutes glutamate stimulation in 14-day neuronal cultures  95

Figure 44  Immunofluorescence image on ERK1/2 activation in response to PP1 inhibition in 14-day neuronal cultures  96

Figure 45  Immunofluorescence image on ERK1/2 activation in response to PP1 inhibition and glutamate stimulation in 14-day neuronal cultures  96

Figure 46  Summary of immunofluorescence experiments on ERK1/2 activation  97

Figure 47  Computational model of the dual effect of PP1 on ERK activity  101

Figure 48  Effects of Ca^{2+} stimulation in the model where NR2B subunits constitute 90% of the total NR2 subunits  106
Figure 49  Effects of Ca\(^{2+}\) stimulation in the model where NR2A subunits constitute 90% of the total NR2 subunits 107

Figure 50  ERK activity after Ca\(^{2+}\) stimulation in the model where NR2B subunits constitute 90% of the total NR2 subunits 108

Figure 51  SynGAP activity after Ca\(^{2+}\) stimulation in the model where NR2B subunits constitute 90% of the total NR2 subunits 109

Figure 52  ERK activity after Ca\(^{2+}\) stimulation in the model where NR2A subunits constitute 90% of the total NR2 subunits 110
LIST OF TABLES

Table 1  Short-term synaptic plasticity model: list of parameters  31
Table 2  Short-term synaptic plasticity model: list of abbreviations  34
Table 3  MAP kinase model: list of parameters  65
Table 4  MAP kinase model: list of abbreviations  76
Table 5  Expanded MAP kinase model: list of additional parameters  102
Table 6  Expanded MAP kinase model: list of abbreviations  105
INTRODUCTION

1.1 Synaptic basis of neuroadaptation

1.1.1 Reward and neuroadaptation

Reward-driven learning is essential for obtaining the resources necessary for survival behaviours such as feeding and reproduction (Hyman et al., 2006). Drugs of abuse are perceived as a kind of reward, and induce persistent compulsive drug intake. Research indicates that neuronal systems can adapt to repeated long-term drug intake, and store drug-associated cues as memory (Berke and Hyman, 2000; Robbins and Everitt, 2002; Everitt and Robbins, 2005; Hyman, 2005). Thus, drug addiction results from the distorted reward-related learning, and from neuroadaptation.

A common feature of reward is the increased dopamine input generated from the ventral tegmental area (VTA) of the midbrain to the nucleus accumbens (NAc) of the ventral striatum (Fig. 1) (Wise, 1998; Chiara, 1998). The NAc, in particular its shell region, is an important target of drugs of abuse (Pontieri et al., 1995; Ito et al., 2004). The NAc also receives glutamatergic afferents from the cerebral cortex, and projects to the pallidum, as part of the cortico-striato-thalamic loop (Parent and Hazrati, 1995). Precise information about specific experiences, cues and actions can be encoded by the induction of bidirectional alterations of synaptic strength, a process termed ‘synaptic plasticity’. In this case, cortico-striatal glutamate inputs and mesencephalic dopamine signals are integrated in the NAc, converting drug associated information and response into long-term memory, via alteration in synaptic plasticity and physical remodelling of the synaptic connections (Berke and Hyman, 2000; Hyman and Malenka, 2001).
1.1 Synaptic basis of neuroadaptation

Figure 1: Dopamine-glutamate interactions in the NAc and the cortico-striato-thalamic loop. This figure illustrates how the glutamate afferent (red line) from the prefrontal cortex (PFC) and dopamine projection (blue line) from ventral tegmental area (VTA) interact at spines in the NAc. The output neurons of NAc, the medium-sized spiny neurons (MSNs), project γ-aminobutyric acid (GABA) (black line) to the ventral analog of the globus pallidus, known as the ventral pallidum (VP). GABAergic VP neurons project to mediodorsal thalamic nucleus (MD), which in turn projects glutamate to PFC.
1.1 Synaptic basis of neuroadaptation

1.1.2 Synaptic plasticity

Neuronal circuits are largely composed of dendrites, axons, and the synapses that connect them, with neuronal information transferred through these connections. A chemical synapse transforms an electrical signal into a chemical one, and consists of a presynaptic active zone, a synaptic cleft and a protein dense region of the spine head called the postsynaptic density (PSD) (Harris et al., 1992). PSD determines the strength of the response to an incoming signal. Bidirectional alterations of this strength are called synaptic plasticity, a process at the origin of learning and memory (Malenka, 2003; Lynch, 2004).

Short term synaptic plasticity

NMDA receptor-dependent long-term potentiation (LTP) and long-term depression (LTD) are two of the most extensively studied prototypic forms of synaptic plasticity. At resting membrane potential, the NMDA receptor is blocked by magnesium. When presynaptic glutamate release coincides with postsynaptic membrane depolarization, the NMDA receptor becomes activatable due to magnesium removal. Glutamate may cause the NMDA receptor to open, and permit calcium ions to flow through the receptor pore (Bliss and Collingridge, 1993; Magee and Johnston, 1997; Markram et al., 1997). The influx of calcium induces the activation of calmodulin, which in turn stimulates the CaMKII (Lisman et al., 2002). The activated CaMKII phosphorylates the GluR1 subunit of the α-amino-3-hydroxy-5-methyl-4-isoxazolepropionic acid (AMPA) receptor (Lee et al., 2000, 2003a), mediating the delivery of AMPA receptors to the postsynaptic membrane (Hayashi et al., 2000; Malinow and Malenka, 2002; Bredt and Nicoll, 2003). This strengthens the pre- and postsynaptic connections, inducing NMDA receptor-dependent LTP (Fig. 2).

Paradoxically, the NMDA receptor-dependent elevation of calcium level is also necessary for the induction of LTD that is characterised by weakened pre- and postsynaptic connections (Mulkey and Malenka, 1992; Kandler et al., 1998). Calcium influx through the NMDA receptor results in the activation of calcineurin and the subsequent activation of PP1, leading to the reduced CaMKII activity and eventually to the rapid reduction in the expression of AMPA receptors in
the postsynaptic membrane (Fig. 2) (Mulkey et al., 1994; Groth et al., 2003).

**Figure 2: NMDA receptor-dependent LTP and LTD.** This figure shows a simplified view of NMDA receptor-dependent short-term synaptic plasticity. **Left:** NMDA receptor-dependent long-term potentiation (LTP). The calcium/calmodulin-dependent protein kinase II (CaMKII)-mediated AMPA receptor insertion into postsynaptic membrane is a major mechanism underlying the initiation of LTP. **Right:** NMDA receptor-dependent long-term depression (LTD). Calcineurin- and protein phosphatase 1 (PP1)-mediated internalisation of postsynaptic AMPA receptor is a primary mechanism of this process.

Therefore, calcium is responsible for triggering both NMDA receptor-dependent LTP and LTD. Surprisingly, however, little is known about the quantitative properties of the postsynaptic calcium signals that can distinguish the activation of these two opposing events.

**Structural plasticity**

Recording memories of neuronal activity is initiated by alterations in glutamate-dependent synaptic transmission. Changes are thereafter stabilised by structural modifications in synaptic connections on dendritic spines. These structural changes, whether dependent on protein synthesis or not, encode persistent long-term memory,
and are referred to as ‘structural plasticity’ (Lamprecht and LeDoux, 2004).

The morphology of spine, the protrusion on dendrites, consists of an expanded head and a neck that connects to the dendritic shaft (Nimchinsky et al., 2002). The excitatory synapse contains the PSD that is developed to efficiently respond to synaptic transmission. Therefore, the shape and number of spines control the characteristics of neurotransmission, and the spines themselves are dynamically regulated by synaptic activity.

Much evidence indicates that induction of synaptic plasticity leads to changes in spine shape (Nikonenko et al., 2002; Yuste and Bonhoeffer, 2001; Fifková and Anderson, 1991), and density (Engert and Bonhoeffer, 1999; Toni et al., 1999; Fiala et al., 2002; Harris et al., 2003). Modifications in spine morphology and number, induced by learning procedures, have also been demonstrated (Leuner et al., 2003; Geinisman et al., 2001; Kleim et al., 2002; Knafo et al., 2001). Spine morphological changes can be observed in as little as two minutes post-stimulation and can last up to months.

The onset of structural change is too rapid to be regulated via nuclear events such as gene expression modifications, or even by direct dendritic protein synthesis. Conversely, it has been shown that more persistent changes require regulation at the level of gene expression and protein translation (Kandel, 2001). Thus, dendritic structural plasticity includes at least two parallel and correlated processes. One process controls the rapid remodelling of cytoskeletal and adhesion proteins (Matus, 2000; Maletic-Savatic et al., 1999), thereby stabilising synaptic connections and consolidating the early phase of synaptic plasticity into late phase. The second process induces the regulation of gene expression and protein synthesis over longer periods of time, consolidating synaptic connections, thus entering the late phase of synaptic plasticity (Fig. 3 and Fig. 4).

The underlying structure of the spine is built upon cytoskeletal filaments composed of actin (Harris and Kater, 1994). Individual actin filaments are transient structures, and filament polymerisations contribute both to the shape of a spine, and to new spine formation (Fischer et al., 1998; Dunaevsky et al., 1999). Indeed, the activation of NMDA receptor, which leads to postsynaptic calcium elevation, has
been shown to induce actin polymerisation (Fukazawa et al., 2003) and to alter spine morphology (Fig. 3) (Maletic-Savatic et al., 1999).

Cytoskeletal rearrangement requires stabilisation, a process that involves the AMPA receptors. These receptors have been demonstrated to maintain spine morphology by sequestering protrusive actin from the surface of the spine head into the core of a spine (Fig. 3) (Fischer et al., 2000). The basal activities of AMPA receptors, induced by spontaneous glutamate release, are able to maintain spine shape in mature synapses (McKinney et al., 1999). Furthermore, the morphological changes of spine, triggered by synaptic activity, can be enhanced by insertion of AMPA receptors into postsynaptic membrane, which maintains LTP (Lamprecht and LeDoux, 2004) and retains memory (Schafe et al., 2001; Lee et al., 2003a; Esteban et al., 2003).

In fact, these morphological changes are correlated with short-term synaptic plasticity. Activation of NMDA receptors induces the activation of CaMKII and the initiation of actin dynamics. Actin-based alterations in spine shape, or the formation of new spines, strengthens synaptic connections. In the meantime, CaMKII facilitates the insertion of AMPA receptors which, in turn, stabilise the actin based structural plasticity. However, similar to the induction of LTD, NMDA receptor-dependent calcium influx can also disrupt the stability of synaptic structure by the activation of calcineurin (Halpain et al., 1998).

The fast rearrangement of the cytoskeleton in spines provides a transition between initial expression of synaptic plasticity and more persistent structural remodelling of dendritic spines via regulation of gene expression and protein synthesis. Protein synthesis can happen locally in dendrite regions proximal to stimulated synapses (Steward and Schuman, 2001; Martin et al., 2000). The synapse-associated polyribosome complexes (SPRCs) are localised precisely at the base of the spine (Steward and Levy, 1982). The mRNA for the CaMKII α subunit is associated with SPRCs, and mRNA levels increase during synaptic activity (Scheetz et al., 2000; Bagni et al., 2000). Many mRNAs encoding different dendritic proteins are locally transcribed in a cell type-dependent manner (Steward and Schuman, 2001). A key advantage of local protein synthesis is that it allows transcription-dependent changes to rapidly occur, which in turn can initiate quickly the late phase of synaptic plasticity.
Figure 3: **Actin-based spine morphology changes and stabilisation.** Synaptic activity induces glutamate release, which activates NMDA receptor, resulting in increased postsynaptic calcium level (*top left*). Rapid actin polymerisation leads to the changes in spine shape (*top right*). The increase of AMPA receptors in the postsynaptic membrane enhances glutamate transmission and stabilises actin filaments (*bottom*).
Sustained structural changes of dendritic spines also require controls at the level of gene expression (Fig. 4). One of the most extensively studied transcription factors, cyclic AMP-response-element-binding protein (CREB), plays a crucial role in establishing and maintaining long-term memory (Kandel, 2001). CREB-mediated gene expression has also been shown to be involved in the long-term effects of drugs of abuse (Blendy and Maldonado, 1998; Carlezon et al., 2005). The targets of CREB include genes encoding brain-derived neurotrophic factor (BDNF), which can lead to structural changes in synapses (Tao et al., 1998; Poo, 2001). Synaptic activity induced phosphorylation and activation of CREB is mediated by calcium-dependent activation of the mitogen-activated protein (MAP) kinase pathway (Wu et al., 2001a; Dolmetsch et al., 2001). However, calcium elevation induced by neuronal activity can also trigger CREB dephosphorylation, through the activation of calcineurin and the subsequent activation of PP1. This NMDA receptor-induced negative regulation of CREB may be affected by the compositional changes of NMDA receptors, and the changes of the associated signalling complexes (Sala et al., 2000; Hardingham et al., 2002).

Long-term exposure to drugs of abuse also leads to large-scale dendritic structural changes, for example, resulting in altered dendritic branching (Robinson and Kolb, 2004). Unlike that of spines, the dendritic cytoplasm is dominated by microtubules (MTs) (Matus, 2000). MTs are in a dynamic state, being constantly polymerised and depolymerised in stable dendrites (Georges et al., 2008). Dendritic branching is initiated by the depolymerisation of MTs and the subsequent MT invasion into filopodia (Georges et al., 2008) that may be newly formed during the induction state of LTP. Many intracellular proteins play roles in regulating dendritic morphology, regulating neurite outgrowth, branching, and stabilisation (reviewed in Arimura and Kaibuchi (2007); Barnes et al. (2008); Barnes and Polleux (2009)).

In summary, structural plasticity due to cytoskeleton rearrangement is rapidly initiated during synaptic activity, followed by a parallel protein synthesis-dependent structural plasticity. This allows both the rapid changes in neurotransmission and the formation of persistent long-term memory.
1.1 Synaptic basis of neuroadaptation

Figure 4: The involvement of gene expression and protein synthesis in spine morphogenesis. Synaptic activity-dependent glutamate release leads to activations of NMDA receptors and second messenger pathways. These result in increased activities of extracellular signal regulated kinases (ERK) and calcium/calcmodulin-dependent protein kinase II (CaMKII) (top left). The fast spine morphogenesis is companioned with the altered nuclear gene expression, mediated by the action of ribosomal protein S6 kinases (RSKs), mitogen- and stress-activated kinases (MSKs), and their substrate cyclic AMP-response-element-binding protein (CREB) (top right). The newly synthesised proteins, for example, AMPA receptors, CaMKII, and actin, promote structural changes of the synapses (bottom).
1.1.3 Signalling pathways responding to dopamine inputs

Dopamine was first shown to be a neurotransmitter in the late 1950s by Carlsson (1959). Since then, dopamine has been found to control many aspects of brain function, including voluntary motor activity, motivation, positive reinforcement and reward-related learning. Because of its importance in those functions, modification of dopamine transmission is also involved in many pathologies such as Parkinson’s disease, schizophrenia, and drug addiction. The dopaminergic neurons involved in reward related learning are located in the midbrain, within the ventral tegmental area and the substantia nigra compacta. They project primarily to the striatum, prefrontal cortex, amygdala, and hippocampus. Dopamine receptors belong to the superfamily of G-protein coupled receptors (GPCRs) and consist of five distinct receptor sub-types in mammals, regrouped in two subfamilies, D1R-like (D1R, D5R) and D2R-like (D2R, D3R, and D4R) (Missale et al., 1998). Dopamine innervation from the midbrain to the striatum exerts its effect mainly through dopamine D1 and D2 receptor subtypes (Berthet and Bezard, 2009). GABAergic medium-sized spiny neuron (MSN), the predominant cell type of the striatum, can be classified into two neuronal populations depending on their projections. MSNs directly projecting to the internal globus pallidus and substantia nigra pars reticulata, and having a stimulatory effect on the cortico-striato-thalamic loop, express mainly dopamine D1 receptors; whereas MSNs projecting to the same nuclei via relays in the external globus pallidus and the subthalamic nucleus, and having an inhibitory effect on the cortico-striato-thalamic loop, express preferentially dopamine D2 receptors (Alexander and Crutcher, 1990). Recent research reveals that different drugs of abuse may exert their long-term effect through different populations of striatal output neurons. For instance, in mouse, acute cocaine injection increases extracellular signal regulated kinases-1/2 (ERK1/2) activity in dopamine D1R-expressing striatonigral neurons (Bertran-Gonzalez et al., 2008).

Dopamine is not an excitatory or inhibitory neurotransmitter, but a neuromodulator. Dopamine exerts effects via adjusting the responses of intracellular signalling pathways to other neurotransmitters, such as glutamate. Because of its positive effect on adenylyl cyclase (AC),
via a olfactory isoform of GTP-binding protein α subunit (Gaolf) in the striatum (Corvol et al., 2001), a major effect of dopamine D1 receptor is to stimulate cyclic adenosine monophosphate (cAMP) production, thus increasing the activation of cyclic AMP-dependent protein kinase (PKA). PKA can directly phosphorylate and activate AMPA receptors to control corticostriatal neurotransmission (Esteban et al., 2003). Dopamine can also act on regulatory proteins that are crucial for the convergence of dopamine- and glutamate-activated signalling cascades, one of which is dopamine- and cAMP-regulated phosphoprotein of 32 kDa (DARPP-32) (Walaas et al., 1983; Greengard, 2001a).

DARPP-32 is highly expressed in MSN (Ouimet et al., 1984, 1998). It is a potent PP1 inhibitor once phosphorylated at Thr34 by PKA (Hemmings et al., 1984a). DARPP-32 interacts with PP1 via two binding domains, both of which contribute to inhibition (Hemmings et al., 1990; Desdouits et al., 1995a; Kwon et al., 1997; Huang et al., 1999). DARPP-32 and inhibitor I (I1) share similar characteristics and, in particular, both inhibit PP1. However, DARPP-32 possesses three other phosphorylation sites, Thr75 by cyclin-dependent protein kinase 5 (CDK5), Ser102 by casein kinase 2 (CK2), and Ser137 by casein kinase 1 (CK1), which make it a signalling hub for kinases and phosphatases. Phosphorylation on Thr75 converts DARPP-32 into an inhibitor of PKA (Bibb et al., 1999). Ser102 phosphorylated DARPP-32 is able to be phosphorylated by PKA more efficiently on Thr34 in vitro (Girault et al., 1989). Furthermore, phosphorylation on Ser137 decreases the rate of Thr34 dephosphorylation by calcineurin (Hemmings et al., 1990; Desdouits et al., 1995b). In short, CK1 and CK2 increase the phosphorylation state of Thr34, thus enhancing D1 dopaminergic signalling and the inhibition of PP1, whereas CDK5 mediates inhibitory effects on dopamine-stimulated signalling pathways. The dephosphorylation of Thr34 is mediated by calcineurin (King et al., 1984) that is activated by calcium and calmodulin (Groth et al., 2003). However, there is evidence indicating that protein phosphatase 2A (PP2A) may also dephosphorylate Thr34, albeit with relatively low efficiency, despite its preferential activity on Thr75 (Nishi et al., 1999). Phospho-Ser137 can be dephosphorylated by protein phosphatase 2C (PP2C) (Desdouits et al., 1998) and Ser102 by PP2A in vitro (Girault et al., 1989). Thus, glutamate-induced activation of calcineurin impairs the
effect of dopamine signalling by releasing DARPP-32 inhibition of PP1. PP2C may produce a synergistic effect with calcineurin by increasing its phosphatase efficiency. The dephosphorylation of Thr34 and Ser102 by PP2A negatively impacts D1 dopaminergic signalling; on the other hand, PP2A effects on Thr75 can enhance dopamine inhibitory effect on PP1 through releasing phospho-Thr75 inhibition on PKA (Fig. 5).

Figure 5: Regulation of DARPP-32 phosphorylation. This figure illustrates the regulation of the four phosphorylation sites of dopamine- and cAMP-regulated phosphoprotein of 32 kDa (DARPP-32) and its inhibitory role on cyclic AMP-dependent protein kinase (PKA) and protein phosphatase 1 (PP1), modified, with permission, from Le Novère et al. (2008). Phosphatases and kinases are represented in red and blue respectively. The graphical conventions are those of the Entity Relationship language of Systems Biology Graphical Notation (SBGN) (Le Novère et al., 2009). Rectangular box with rounded corners: entity, circle on entity: state, circle not on entity: state value, harpoon arrow: assignment, black dot: outcome, empty arrow head: stimulation, bar arrow head: inhibition.

An additional layer of complexity comes from the intricacy of the regulation of these kinases and phosphatases. The cAMP-regulated PKA phosphorylates and increases the phosphatase activity of PP2A. PP2A dephosphorylates DARPP-32 on Thr75 and thereby forming a positive feedback loop on dopamine signal efficiency (Nishi et al.,
However, PKA can also activate cyclic nucleotide phosphodiesterase (PDE), which in turn catalyses the degradation of cAMP, reducing the duration of dopamine signal transmission. Glutamate induced calcium elevation not only activates calcineurin but also PP2A, enhancing dopamine signal efficiency via dephosphorylation of Thr75 (Nishi et al., 2002; Ahn et al., 2007b). Furthermore, DARPP-32 can translocate to the nucleus upon dephosphorylation at Ser102 by PKA-dependent activation of PP2A. Thus, drugs of abuse cannot only induce DARPP-32 phosphorylation on Thr34 but also cause rapid nuclear accumulation, resulting in PP1 inhibition in the nucleus. PP1 has very broad range of substrates, including promoting the histone H3 phosphorylation, thus controlling nuclear function and long-term drug effect (Stipanovich et al., 2008).

Therefore, DARPP-32 cannot be simply considered as a switch that either inhibits PP1 or PKA, corresponding to dopamine or glutamate transmission. More precisely, DARPP-32 acts as an integrator, combining the activities of enzymes regulated by dopamine and/or glutamate into graded inhibition of PP1 and PKA, in a temporal and spatial manner (Le Novère et al., 2008). DARPP-32 plays a very important role in dopamine induced phosphorylation of NMDA and AMPA receptors, and the induction of LTP and LTD in striatal neurons (Yan et al., 1999; Fienberg et al., 1998; Snyder et al., 1998; Flores-Hernández et al., 2002; Calabresi et al., 2000). Besides, DARPP-32 modulates dopamine D1 receptor-induced gene transcription (Svenningsson et al., 2000), which is important for initiating drug-induced long-term adaptive morphological changes. All in all, dopamine, via DARPP-32, regulates the potency of PKA and PP1 to control the plasticity of corticostriatal connections and long-term neuroadaptation of basal ganglia.

1.2 Modelling signalling pathways in neurons

Neurons can process and store information through the activation of biochemical signalling networks. Neurotransmitters released in synapses can activate membrane receptors, where these activations result in increased or decreased second messenger levels, which themselves can affect many signalling pathways. These pathways are densely
First of all, neuronal signalling is based on precise timing. The activation of NMDA receptor requires the binding of glutamate to coincide with postsynaptic membrane depolarization, which then allows calcium influx, triggering specific neuronal responses (Bliss and Collingridge, 1993; Magee and Johnston, 1997; Markram et al., 1997). In reward-related learning, the reward signal-inducing dopamine release must pair with the sensory-motor input-triggering glutamate release, so that dopamine modulates glutamate signalling pathways and increases the expression of molecules involved in learning (Centonze et al., 2001; Wickens et al., 2003).

Secondly, neuronal biochemical networks are able to distinguish signals with temporal patterns (Bhalla, 2003). For instance, central to this thesis are the different frequencies of calcium spikes triggering distinct responses, including calcineurin and CaMKII, that results in the induction of LTD or LTP.

Thirdly, signalling proteins, especially the ones located in the vicinity of the postsynaptic membrane, are assembled and organised in a way that allows the initiation of selective modulations of synaptic activity (Kennedy, 2000). One example is the NMDA receptor signalling complex. The cytoplasmic tail of the NR2 subunit of NMDA receptor serves as a scaffold protein, associating many crucial kinases and phosphatases, including CaMKII and synaptic Ras GTPase activating protein (SynGAP). These will be explored in more detail later in this thesis.

Last, but not least, the characteristic activities of neuronal signalling networks span multiple timescales. Fast synaptic transmission occurs in less than 1 millisecond, whereas slow transmission takes place over periods of hundreds of milliseconds to minutes (Greengard, 2001b). Glutamate-induced MAP kinase activation can last for hours, whereas changes in neuronal morphology may require days.

These properties are difficult to control and manipulate during in vivo experiments. Therefore we need to build quantitative computational models in order to interpret the results from molecular experiments, to predict the consequences of our various hypotheses, and
finally to map and gain a system-level understanding of the different relevant pathways.

Several mathematical approaches are available for representing and analysing neuronal biochemical signalling, which can be classified into three major categories (reviewed by Eungdamrong and Iyengar (2004) and Tölle and Le Novère (2006)): deterministic simulations of continuous variables, population based stochastic simulations and particle based stochastic approach.

1.2.1 Deterministic simulations of continuous variables

Simulations of deterministic systems do not involve any stochastic considerations. Therefore, the changes in concentrations of components over time are predictable if the initial conditions and boundaries are specified.

Continuous models based on homogeneous systems

Most deterministic simulations assume that chemical reactions happen in well-stirred compartments, and models are therefore built on continuous ordinary differential equation (ODE) systems. Typical examples are kinetic models based on Mass Action kinetics (Guldberg and Waage, 1864), which relate the velocity of an elementary reaction to the product of the concentrations of its reactants, to the powers of their respective stoichiometries. The speed at which a component changes depends on the rates of reactions at which this component is consumed or produced.

This kind of modelling is computationally efficient, and many ODE solvers have been implemented. If each chemical species in the model is present in a large quantity, these homogeneous and continuous ODE systems can provide a good approximation of the behaviour of a large network (for example, see Bhalla and Iyengar (1999)). Since these models can be efficiently simulated, they are amenable to large-scale parameter scanning for initial concentrations and kinetic constants, therefore providing useful insights on parameter constraints for the behaviours of the relevant signalling pathways (see for example Fernandez et al., 2006)).
Deterministic models based on heterogeneous systems

Space homogeneity is not always the reality. For example, neuronal activity is often regulated by localised constructs, as demonstrated by the signalling complexes within the PSD. Often, a more realistic approach is to consider multiple compartments in order to emulate concentration gradients, different cellular organelles, and scaffolds-based signalling complexes. In these instances, the diffusion of molecules between the different compartments can be modelled as transport reactions. Within each compartment, molecules may be considered as being distributed homogeneously, thus compartmentalised interactions can still be modelled by continuous ODE systems (for example, see Naoki et al. (2005)). However, when a chemical component appears in different compartments, each compartment needs to be treated as an independent pool, thus increasing the complexity of the model. Furthermore, the rates of diffusion between compartments are usually only roughly estimated.

Another realistic approach is to use reaction-diffusion equations based on partial differential equations (PDE). In this modelling approach, the concentration change of a component depends not only on the net rate of consuming and producing reactions, but also on its diffusion in space. Therefore, the concentration changes depend on multiple independent variables, including time and the location of the molecule in the concentration gradients. To numerically solve reaction-diffusion equations requires not only the initial conditions, but also the concentrations at the boundaries of the specified compartments (Fall et al., 2002) (for examples, see Fink et al. (2000), Haugh (2002), and Shvartsman et al. (2002)). The drawbacks of this modelling approach are increased complexity and computing time. Moreover, the values for diffusion constants in vivo are generally unavailable.

1.2.2 Population-based stochastic simulations

Biochemical components participate in reactions as integer numbers, which define the discrete nature of a biological system. Many biochemical activities do not follow predicted trajectories when the initial conditions are specified. This is due to the noise and randomness
in neuronal signalling, both of which are unavoidable because of reactions occurring in space-restricted environments, and because of the small numbers of each species involved. In this case, the changes in concentration of components over time cannot be precisely predicted, and continuous deterministic approaches may lead to inaccurate results. Probabilistic descriptions of chemical reactions are therefore preferred.

A family of widely used algorithms for simulating stochastic biochemical models was proposed by Gillespie (1976, 1977), based on the kinetic Monte Carlo Method (Bortz et al., 1975). In Gillespie-type methods, reactions occur with certain probabilities defined by their kinetic rates and the number of molecules present. Random numbers are used to decide the delay before the next step, and which reactions will occur (for examples, see Bhalla (2004a) and Bhalla (2004b)). Stochastic simulation approaches can also be incorporated within a spatially heterogeneous reaction-diffusion model (Lee et al., 2003b).

In many circumstances, however, reactions with small numbers of reactants coexist with reactions involving great numbers of signalling molecules. An ideal approach, in terms of speed and accuracy, would be to use a hybrid system incorporating both stochastic and deterministic simulation methods, as demonstrated in Tian et al. (2007).

1.2.3 Particle-based stochastic approach

All simulation approaches described above treat reacting components as population pools. One problem arising from this implementation is the potential combinatorial explosion resulting from multi-state molecules (reviewed by Tölle and Le Novère (2006)). An attractive alternative is to use particle-based stochastic approaches, in which each individual molecule is treated as an object. This allows understanding of the contribution of each individual particle within the whole system. Another advantage of a particle-based approach is that positional information, orientation, and geometry for each molecule can be monitored, which has significant effects in the study of spatially heterogeneous system. However, this also means that additional parameters need to be obtained or estimated (for example, see Franks et al. (2002) and Franks et al. (2003)).
2.1 Introduction

NMDA receptor-dependent LTP and LTD are two forms of activity-dependent synaptic plasticity, a process at the origin of learning and memory (Malenka, 2003; Lynch, 2004). It has been shown that a high-frequency of synaptic stimulation leads to LTP (Bliss and Collingridge, 1993), while a low-frequency stimulation results in LTD (Dudek and Bear, 1992). In both cases, stimulation triggers postsynaptic membrane depolarization, which leads to the activation of synaptic NMDA receptors, and the subsequent elevation of intracellular calcium concentration. Calcium, via calmodulin, activates the CaMKII, inducing LTP (McGlade-McCulloh et al., 1993; Lledo et al., 1995), or calcineurin, triggering LTD (Mulkey et al., 1994).

It has been proposed that substantial increases in postsynaptic calcium concentration selectively activate CaMKII, while moderate rises activate calcineurin (PP2B) (Artola and Singer, 1993; Lisman, 1989). However, several observations suggest this hypothesis is inadequate. First of all, intracellular calcium level increases in the form of spikes rather than by gradually reaching a steady level. This is due to the large number of calcium-binding proteins, which acts as the calcium buffer, and to calcium efflux mechanisms, which lower calcium concentration back to a basal level within a few hundred milliseconds (Sabatini et al., 2002). This fast calcium transient suggests that the increase in calcium level depends not only on the amplitude of each input, but also on the frequency and duration of these inputs. Second, a brief sub-molar increase of calcium has been found to trigger LTP and LTD with similar probabilities (Neveu and Zucker, 1996). Third, the different temporal patterns of postsynaptic calcium elevation have been shown to selectively induce LTP or LTD (Yang et al., 1999). Taken together, these findings suggest that the temporal patterns of calcium increase, rather than its amplitude, are the key signal carrying sig-
significant biological information. The question remains, however, as to how signalling pathways are able to decipher the temporally-encoded calcium signals through key signalling molecules such as calmodulin, or CaMKII.

Calmodulin, an important calcium-dependent regulatory protein, possesses four EF-hand calcium-binding domains (Babu et al., 1988), and can exist in two distinct conformations: the inactive T state (Kuboniwa et al., 1995) and the active R state (Babu et al., 1985). Calcium cooperatively binds to calmodulin (Crouch and Klee, 1980). The binding of four calcium ions is not necessary for calmodulin function, since unsaturated calmodulin can also activate its targets (Shifman et al., 2006). Stefan et al. (2008) proposed an allosteric model for calmodulin activation, illustrating how the binding of calcium ions progressively stabilises the high-affinity R state. Furthermore, in this model, calmodulin can differentially activate calcineurin and CaMKII according to static calcium concentration values. This raises the question of whether this allosteric device is also able to decode patterns of calcium spikes.

CaMKII is a crucial mediator for NMDA receptor-dependent synaptic plasticity (Lisman et al., 2002; Malenka and Nicoll, 1999). CaMKII holoenzyme is a dodecamer structure composed of two rings of hexamers (Rosenberg et al., 2005; Kolodziej et al., 2000). Binding of calmodulin activates the CaMKII monomer (Hanley et al., 1988), and initiates its autophosphorylation on threonine 286 (Thr286). This autophosphorylation requires the catalytic ability from an active neighbour subunit within the hexameric ring (Hanson et al., 1994; Mukherji and Soderling, 1994). Phosphorylated CaMKII remains in a constant active state, independent of calmodulin activity (Payne et al., 1988; Hanson et al., 1994). This autophosphorylation also increases the apparent affinity of calmodulin binding to CaMKII (Meyer et al., 1992). CaMKII autophosphorylation has been shown to be a decoder of calcium spike frequency in vitro (De Koninck and Schulman, 1998). Many questions, however, remain open. How does CaMKII respond to calcium inputs in the presence of phosphatases? What is the relative activity of CaMKII and calcineurin regulated during high-frequency calcium oscillations?
Calcineurin exerts its effect on synaptic plasticity in MSN by dephosphorylating DARPP-32 on Thr34 (Hemmings et al., 1984a), relieving inhibition on PP1, thereby dephosphorylating CaMKII on Thr286 (Strack et al., 1997a). The fact that PP1 inhibition blocks the induction of LTD demonstrates the important modulatory roles played by inhibitors of PP1 (Kirkwood and Bear, 1994; Mulkey et al., 1993, 1994). Drugs of abuse cause extracellular dopamine increase, which enhances DARPP-32 phosphorylation on Thr34, and PP1 inhibition. Then, how drugs of abuse affect CaMKII activity?

To answer these questions, detailed quantitative computational models are required, since for example, biochemical experiments are often constrained by the limitations of available chelators able to produce different patterns of calcium spikes (Yang et al., 1999). A few models have already investigated the effect of calcium on postsynaptic plasticity. For instance, based on a spatial model, Franks et al. (2001) showed that calmodulin activation depends on the frequencies of calcium spikes, and this response to frequency can be modulated by the availability of calmodulin and calcium binding proteins. Bhalla (2002) demonstrated that signalling pathways can discriminate between eight complex calcium input patterns, and raise distinct activities of kinases and phosphatases. D’Alcantara et al. (2003) provided insight into how a calcium increase of high amplitude and short duration could induce AMPA receptor activation; while a low amplitude, long duration signal triggers AMPA receptor inactivation. Naoki et al. (2005) studied how different spike frequencies and shapes of calcium can influence calmodulin activation in the area near calcium channels, especially when calmodulin is exchangeable to other local regions within the dendritic spine. Urakubo et al. (2008) set up an allosteric model of NMDA receptor and showed that presynaptic glutamate release followed by postsynaptic membrane depolarization induces activation of NMDA receptors and large postsynaptic calcium spikes, resulting in LTP. Pepke et al. (2010) revealed that calmodulin with fewer than four bound calcium ions and its sensitivity to calcium spike frequency contribute primarily to CaMKII activation. However, none of these models accurately modelled calmodulin activation in the first place, nor considered CaMKII autophosphorylation in the context of a hexamer. Besides, the correlation between the frequency
of calcium inputs and the total amount of calcium ions has never been systematically studied before.

In the present study, I used an existing model of calmodulin developed by Stefan et al. (2008) based on the allosteric framework. The model accurately reproduces the activation of calmodulin by calcium as observed experimentally. I expanded this model to include CaMKII autophosphorylation, using a rate based on the probability of having an active neighbour subunit, at each simulation step. Reactions describing the dephosphorylation of CaMKII by PP1, the inhibition of PP1 by DARPP-32, and the dephosphorylation of DARPP-32 by calcineurin were included in addition (as shown in Fig. 6). A detailed calcium buffering system was embedded, with each calcium spike accurately modelled and in accord with published experimental data (Sabatini et al., 2002). The effects of a wide range of calcium frequencies, amplitudes, and durations were studied. The activation of calcineurin and CaMKII were compared, based on the amplitudes and durations of their activation.

In the following sections, I show that the activation of calmodulin depends on the frequency of calcium inputs. With a total amount of calcium ions kept constant, high frequency pulses stimulate calmodulin more efficiently. Calcium inputs activate both calcineurin and CaMKII at all frequencies, but increased frequencies shift the balance of relative activation from calcineurin to CaMKII. Independent of the input amplitude and duration, the total amount of calcium ions injected adjusts the sensitivity of the system towards calcium-input frequencies. At a given frequency, the quantity of CaMKII activated is proportional to the total amount of calcium. Thus, a small amount of calcium input at high frequency can induce the same activation of CaMKII as observed for larger amounts, at lower frequencies. Finally, the extent of activation of CaMKII at high calcium signal frequency is further controlled by other factors, including by the availability of calmodulin, and the potency of phosphatase inhibitors.
Figure 6: Reaction diagram of calmodulin regulated pathways. Graphical representation of the model implemented in this study. Part of this model is based on a published allosteric model of calmodulin function (Stefan et al., 2008). Only the extension is displayed here. Filled arrow: yield, bar arrow: inhibition, circle: catalyse, CaMR: calmodulin in active state, PP2B: calcineurin. CaMKII: calcium/calmodulin-dependent protein kinase II. PP1: protein phosphatase 1. PKA: cyclic AMP-dependent protein kinase. DARPP-32: dopamine- and cAMP-regulated phosphoprotein of 32 kDa.
2.2 Modelling methods

2.2.1 Modelling and simulation software

Models were based on the description of biochemical processes using continuous variables, that were simulated with a deterministic method. Modelling and simulation were performed using the E-cell system, version 3 (Takahashi et al., 2004). E-Cell system is a module based, object-oriented simulation environment suitable for modelling, simulating and analysing of large-scale cell biological models. E-Cell defines the simulation model as a set of objects connected to each other. A biochemical reaction can be built by connecting variable, process, and stepper objects, where a variable represents a molecular species (an entity pool), a process represents the kinetic law that results in changes in the values of several variables, and a stepper attaches a specific simulation algorithm to a set of processes. E-cell supports various stepper functions, and can incorporate different algorithms and time scales into one model via its unique discrete integration meta-algorithm.

A generic ODEStepper, based on the combined Radau 5 and Dormand-Prince 5(4)7M (Dormand and Prince, 1980) algorithms, was used for the elementary reactions. E-Cell switches between these two algorithms depending on the stiffness of the system of equations at a given time point. Simulations were performed on the computing cluster of the European Bioinformatics Institute, which is composed of Intel-based nodes under GNU/Linux. The simulations were launched concurrently with the calcium firing at one frequency running on an individual core.

2.2.2 Reaction and parameters

My models presented in this thesis were based on a single homogeneous compartment, with a volume of $10^{-15}$ L representing the spine (Nimchinsky et al., 2002). Reactions were modelled with Mass Action Law (MAL) processes. Each enzymatic reaction was represented by each of the three elementary steps for binding (1), dissociation (2), and catalysis (3). Michaelis-Menten kinetics were not used for
2.2 Modelling methods

enzymatic reactions, since it assumes substrate concentration greatly exceeds the enzyme concentration. Based on this assumption, an enzyme may be characterised as an independent catalyst, and the concentration of an enzyme-substrate complex may always be assumed to be constant (quasi-steady state approximation) (Briggs and Haldane, 1925). However, when the system contains similarly distributed enzymes and substrates, the differences between the macro-level (e.g. Michaelis-Menten kinetics) and the micro-level descriptions (e.g. reaction rate for elementary steps) should not be neglected (Kholodenko and Westerhoff, 1995). Thus, the elemental step description is always a more precise way to gain an accurate and quantitative understanding of enzyme kinetics.

\[
\begin{align*}
E + S & \xrightleftharpoons[k_{\text{off}}]{k_{\text{on}}} ES \\
ES & \xrightarrow{k_{\text{cat}}} E + P
\end{align*}
\]

The association \((k_{\text{on}})\), dissociation \((k_{\text{off}})\), and catalytic \((k_{\text{cat}})\) constants were mainly obtained from published kinetic constants, retrieved from the Database Of Quantitative Cellular Signalling (http://doqcs.ncbs.res.in/), or estimated.

2.2.3 Pathway activation

Ca\(^{2+}\) inputs were implemented using Python scripts, consisting of repeated increases of the calcium influx constant of a zero-order reaction in the model. The duration for each increase was 8 milliseconds. Throughout most of the study, the number of calcium ions inserted in the system at each input was 43200 molecules (that is the product of influx rate multiplied by duration). To study the effect of input size, the number of calcium ions for each input was increased, by increasing the influx rate. The delay before the next calcium increase varied according to the frequency. However, when the input frequency was too high to have the time interval between two spikes longer than 8 milliseconds, the influx constant and opening duration would be recalculated in order to keep the number of calcium ions in each input constant. For instance, a 50 Hz signal was composed of a train of calcium inputs, each of which lasted 8 milliseconds, and a 12 milliseconds interval between each pair of inputs. However, at 200 Hz, the
delay between each opening was 5 milliseconds. Since this was below the 8 milliseconds threshold, the system considered the opening was 5 milliseconds, and calculated the new influx constant based on this new opening time.

2.3 Postsynaptic calcium affects the ratio of calcineurin and CaMKII activation

2.3.1 Model structure

The activation of calmodulin by calcium was modelled as described previously (Stefan et al., 2008). In the following sections, I first introduce this existing calmodulin model. The extension of this model is explained afterwards.

Allosteric model of calmodulin by Stefan et al.

This model describes two states of calmodulin, the active (R) and the inactive (T) state. In either state, calmodulin can bind up to four calcium ions, with calcium affinity being higher for R state than for T state. Each calcium-binding site is considered unique and modelled explicitly, with its own specific dissociation constant. Calmodulin, regardless of the number of calcium ions bound, can undergo transitions between R and T states. The binding of calcium progressively lowers more the free energy of the R state than that of T state. As a consequence, calcium ions cooperatively bind to calmodulin, facilitating the transition from T to R state. Once calmodulin is in the R conformation, it can bind to target proteins, calcineurin and CaMKII in the model, and can activate them (Fig. 7).

The allosteric isomerisation constant L \( (L = \frac{[T_0]}{[R_0]}) \) and the ratio of R and T affinity for each site \( (c_i = \frac{K^R_i}{K^T_i}) \) are estimated by assuming all four \( c_i \) values are identical and all four binding sites are equivalent. Then, the experimental data of calcium-binding in the presence and absence of an allosteric activator for mutant and wild-type forms of calmodulin (Peersen et al., 1997), is fitted in the equation describing the fractional occupation correspondingly (Rubin and Changeux, 1966) to obtain L and c values. Then the microscopic dissociation constants for R state \( (K^R_i) \) are estimated by fitting experimen-
tal data of calmodulin binding to calcium under different experimental conditions (Bayley et al., 1996; Shifman et al., 2006; Peersen et al., 1997), into a generalised equation for fraction occupation of calmodulin with non-equivalent calcium-binding sites (Monod et al., 1965).
Figure 7: Allosteric model of calmodulin by Stefan et al. Graphical representation of the published allosteric model of calmodulin (Stefan et al., 2008). The four binding sites of calmodulin are designated as A, B, C, and D. Filled arrow: yield, CaMT: calmodulin in inactive state, CaMR: calmodulin in active state, PP2B: calcineurin. CaMKII: calcium/calmodulin-dependent protein kinase II.
2.3 Calcium controls the relative activation

*Extension of the model of calmodulin*

The model described above was extended by including a detailed description of the processes controlling CaMKII autophosphorylation. Only monomers of CaMKII were considered and the autophosphorylation was modelled in a first order reaction upon calmodulin binding. In order to accurately model the autophosphorylation on Thr286 within the context of the hexamer and take into account the fact that this is a trans-phosphorylation between adjacent subunits (Hanson et al., 1994), I computed a correction of the rate based on the probability that monomers have active neighbours. I proceeded in two steps. On the one hand, for each number of active monomers per hexamer, I computed the probability for a given subunit of having an active neighbour. On the other hand, I set up a random simulator to calculate the distributions of active CaMKII subunits within hexamers as a function of the total amount of active subunits (http://www.ebi.ac.uk/~luli/thesis/calcium/). For a given quantity of active CaMKII monomers, I randomly picked the location of active monomers within the hexamers, and recorded the number of hexamers containing a specific number of active subunits. After 1000 repeats of this random simulation, the average fraction of each number of active monomers per hexamer was multiplied by its corresponding probability of having an active neighbour computed in the first step. The sum of these six numbers was then used as a coefficient to adjust the autophosphorylation rate. The whole procedure was repeated for every 1% increase of active monomers. Finally, these 100 generated values were fitted by a polynomial function of degree 5. This polynomial function was embedded in the model for E-Cell, dynamically changing the autophosphorylation rate (the detailed procedure for this calculation is illustrated in Fig. 8, the fitted polynomial function is plotted in Fig. 9). Once CaMKII was phosphorylated, its affinity for calmodulin increased in the model, according to Meyer et al. (1992).
Calcium controls the relative activation of CaMKII. Figure 8: Calculation the rate of CaMKII autophosphorylation. This figure shows the procedure for calculating the correction of the autophosphorylation rate as a function of active CaMKII monomers.
2.3 Calcium controls the relative activation

The catalytic subunit of calcineurin (calcineurin A or CNA) is activated upon association with Ca\textsuperscript{2+}/calmodulin (Klee et al., 1998). Calcineurin has a relatively higher affinity for calmodulin binding than other calmodulin-binding proteins (Quintana et al., 2005; Perrino et al., 2002) and, in particular, CaMKII, which facilitates its sensitive response to calcium-mediated synaptic stimulation (Groth et al., 2003). Activated calcineurin dephosphorylates DARPP-32 on Thr\textsubscript{34} (Hemmings et al., 1984a).

PP1 is a major eukaryotic protein serine/threonine phosphatase that regulates diverse cellular processes, including the dephosphorylation of CaMKII and the induction of LTD (Mulkey et al., 1994). The catalytic subunit of PP1 is inhibited by DARPP-32 upon its phosphorylation on Thr\textsubscript{34} by PKA (Hemmings et al., 1984a; Svenningsson et al., 2004). In this model, approximately 40\% of DARPP-32 was activated by the basal level of PKA. The phosphorylated DARPP-32, in turn, inhibited almost 90\% of the total PP1. The inhibition was

![Fitting curve](image)

**Figure 9:** The fitted polynomial function for the rate of CaMKII autophosphorylation. The polynomial function was fitted by 100 corrections of the rate of CaMKII autophosphorylation. The 100 corrections were calculated in terms of the distribution of activated CaMKII monomer and the probability of having an active neighbour.
modelled following a competitive inhibition mechanism. Only phosphorylation on Thr34 of DARPP-32 was considered in this model.

The calcium efflux was modelled assuming a single clearance process, using Michaelis-Menten kinetics. The calcium influx was modelled through a zero-order reaction with a fixed rate. The vast number of calcium buffer proteins, with different dissociation constants were adopted in order to provide an efficient calcium reservoir (Naoki et al., 2005).

All the quantitative parameters used in this model are listed in Table 1, and abbreviations are explained in Table 2. This model and the Python script used for simulation in E-Cell are available online (http://www.ebi.ac.uk/~luli/thesis/calcium/).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ca^{2+}$ binding calmodulin (CaM):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{\text{ion}}$</td>
<td>$10^6 \text{ M}^{-1}s^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{off}_A}$</td>
<td>$8.32 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{off}_B}$</td>
<td>$1.66 \times 10^{-2} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{off}_C}$</td>
<td>$17.4 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{off}_D}$</td>
<td>$1.45 \times 10^{-2} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{R}_{A}}$</td>
<td>$2.10 \times 10^{3} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{R}_{B}}$</td>
<td>$4.19 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{R}_{C}}$</td>
<td>$4.39 \times 10^{3} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{R}_{D}}$</td>
<td>$3.66 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{R}}$</td>
<td>$1 \times 10^{6} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{TR}}$</td>
<td>$48.38 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{Ca}}$</td>
<td>$6.2829 \times 10^{4} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{Ca}}$</td>
<td>$768.81 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{Ca}}$</td>
<td>$3.96 \times 10^{3} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{Ca}}$</td>
<td>$1.2217 \times 10^{4} \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
<tr>
<td>$k_{\text{CaM}_{Ca}}$</td>
<td>$249.2 \text{ s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*,[1]$</td>
</tr>
</tbody>
</table>
Table 1: continued

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{CaM3Ca}^{TR}$</td>
<td>$1.94144 \times 10^5 \text{s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*$,$^1$</td>
</tr>
<tr>
<td>$k_{CaM4Ca}^{TR}$</td>
<td>15.6816 s$^{-1}$</td>
<td>(Stefan et al., 2008)$^*$,$^1$</td>
</tr>
<tr>
<td>$k_{CaM4Ca}^{TR}$</td>
<td>$3.085144 \times 10^6 \text{s}^{-1}$</td>
<td>(Stefan et al., 2008)$^*$,$^1$</td>
</tr>
</tbody>
</table>

Ca$^{2+}$ binding calcium buffer protein (CBP):

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on_{CBPfast}}$</td>
<td>$10^9 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{off_{CBPfast}}$</td>
<td>$10^3 \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{on_{CBPmedium}}$</td>
<td>$10^8 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{off_{CBPmedium}}$</td>
<td>$10^2 \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{on_{CBPslow}}$</td>
<td>$10^7 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{off_{CBPslow}}$</td>
<td>$10 \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{on_{CBPvslow}}$</td>
<td>$10^6 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
<tr>
<td>$k_{off_{CBPvslow}}$</td>
<td>$1 \text{s}^{-1}$</td>
<td>(Markram et al., 1998)$^*$,$^2$</td>
</tr>
</tbody>
</table>

Ca$^{2+}$ pump:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{max}$</td>
<td>$4 \times 10^{-3} \text{Ms}^{-1}$</td>
<td>(cf. Markram et al., 1998)$^*$,$^3$</td>
</tr>
<tr>
<td>$K_m$</td>
<td>$1 \times 10^{-6} \text{M}$</td>
<td>(Markram et al., 1998)$^*$</td>
</tr>
</tbody>
</table>

Ca$^{2+}$ leak:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$4 \times 10^{-5} \text{Ms}^{-1}$</td>
<td>(cf. Markram et al., 1998)$^*$,$^3$</td>
</tr>
</tbody>
</table>

Ca$^M^R$ binding substrates:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on_{CaMKII}}$</td>
<td>$3.2 \times 10^6 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Tzortzopoulos and Török, 2004)$^4$</td>
</tr>
<tr>
<td>$k_{off_{CaMKII}}$</td>
<td>0.343 \text{s}^{-1}</td>
<td>(Tzortzopoulos and Török, 2004)$^4$</td>
</tr>
<tr>
<td>$k_{on_{CaMKIIp}}$</td>
<td>$3.2 \times 10^6 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Tzortzopoulos and Török, 2004)$^4$</td>
</tr>
<tr>
<td>$k_{off_{CaMKIIp}}$</td>
<td>0.001 \text{s}^{-1}</td>
<td>(Meyer et al., 1992)$^5$</td>
</tr>
<tr>
<td>$k_{off_{PP2B}}$</td>
<td>$4.6 \times 10^7 \text{M}^{-1} \text{s}^{-1}$</td>
<td>(Quintana et al., 2005)$^6$</td>
</tr>
<tr>
<td>$k_{off_{PP2B}}$</td>
<td>0.4 \text{s}^{-1}</td>
<td>(Perrino et al., 2002)$^7$</td>
</tr>
</tbody>
</table>

CaMKII autophosphorylation on Thr286:
Calcium controls the relative activation

Table 1: continued

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{Thr286p}$</td>
<td>$6.3 \text{s}^{-1}$</td>
<td>(Lučić et al., 2008)(^8). This parameter is further refined during simulation run.</td>
</tr>
</tbody>
</table>

**PKA phosphorylates DARPP-32 on Thr34:**

- $k_{\text{on}_{D,\text{PKA}}}$: $5.6 \times 10^6 \text{M}^{-1}\text{s}^{-1}$ (Hemmings et al., 1984b)\(^9\)
- $k_{\text{off}_{D,\text{PKA}}}$: $10.8 \text{s}^{-1}$ (Hemmings et al., 1984b)\(^9\)
- $k_{\text{cat}_{D,\text{PKA}}}$: $2.7 \text{s}^{-1}$ (Hemmings et al., 1984b)\(^9\)

**Calcineurin (PP2B) dephosphorylates DARPP-32 on Thr34:**

- $k_{\text{on}_{D,\text{PP2B}}}$: $4.1 \times 10^6 \text{M}^{-1}\text{s}^{-1}$ (King et al., 1984)\(^10\)
- $k_{\text{off}_{D,\text{PP2B}}}$: $6.4 \text{s}^{-1}$ (King et al., 1984)\(^10\)
- $k_{\text{cat}_{D,\text{PP2B}}}$: $0.2 \text{s}^{-1}$ (King et al., 1984)\(^10\)

**DARPP-32Thr34p binding PP1:**

- $k_{\text{on}_{D,\text{PP1}}}$: $4.0 \times 10^6 \text{M}^{-1}\text{s}^{-1}$ (Desdouits et al., 1995a)\(^11\)
- $k_{\text{off}_{D,\text{PP1}}}$: $0.4 \text{s}^{-1}$ (Desdouits et al., 1995a)\(^11\)

**PP1 dephosphorylates CaMKII:**

- $k_{\text{on}_{\text{CaMKII}_p,\text{PP1}}}$: $3.0 \times 10^6 \text{M}^{-1}\text{s}^{-1}$ this study\(^*,12\)
- $k_{\text{off}_{\text{CaMKII}_p,\text{PP1}}}$: $0.5 \text{s}^{-1}$ this study\(^*,12\)
- $k_{\text{cat}_{\text{CaMKII}_p,\text{PP1}}}$: $2.0 \text{s}^{-1}$ (Zhabotinsky, 2000)\(^*\)

**Concentrations:**

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{CaM}^T]_0$</td>
<td>$3 \times 10^{-5} \text{M}$</td>
<td>(Kakiuchi et al., 1982)(^13)</td>
</tr>
<tr>
<td>$[\text{Ca}^{2+}]_{\text{basal}}$</td>
<td>$1 \times 10^{-8} \text{M}$</td>
<td>(Allbritton et al., 1992)(^14)</td>
</tr>
<tr>
<td>$[\text{PP2B}]$</td>
<td>$1.6 \times 10^{-6} \text{M}$</td>
<td>(cf. Goto et al., 1986)(^15)</td>
</tr>
<tr>
<td>$[\text{CaMKII}]$</td>
<td>$7 \times 10^{-5} \text{M}$</td>
<td>(cf. Petersen et al., 2003)(^16)</td>
</tr>
<tr>
<td>$[\text{DARPP-32}]$</td>
<td>$3 \times 10^{-6} \text{M}$</td>
<td>(estimated from Halpain et al., 1990)(^17)</td>
</tr>
<tr>
<td>$[\text{PKA}]$</td>
<td>$1.2 \times 10^{-8} \text{M}$</td>
<td>(estimated from Bacskaï et al., 1993)(^18)</td>
</tr>
<tr>
<td>$[\text{PP1}]$</td>
<td>$2 \times 10^{-6} \text{M}$</td>
<td>(cf. Ingebritsen et al., 1983)(^19)</td>
</tr>
<tr>
<td>$[\text{CBP}_{fast}]$</td>
<td>$8 \times 10^{-5} \text{M}$</td>
<td>(Naoki et al., 2005)(^*)</td>
</tr>
<tr>
<td>$[\text{CBP}_{medium}]$</td>
<td>$8 \times 10^{-5} \text{M}$</td>
<td>(Naoki et al., 2005)(^*)</td>
</tr>
</tbody>
</table>
2.3 Calcium controls the relative activation

Table 1: continued

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaM</td>
<td>calmodulin</td>
</tr>
<tr>
<td></td>
<td>UniProt:P62158</td>
</tr>
<tr>
<td>PP2B</td>
<td>calcineurin/protein phosphatase 2B</td>
</tr>
<tr>
<td></td>
<td>GO:0005955</td>
</tr>
<tr>
<td>CaMKII</td>
<td>calcium/calmodulin-dependent protein kinase II</td>
</tr>
<tr>
<td></td>
<td>GO:0005954</td>
</tr>
<tr>
<td>DARPP-32</td>
<td>dopamine- and cAMP-regulated phosphoprotein of 32 kDa</td>
</tr>
</tbody>
</table>

Table 2: List of abbreviations used in the short-term synaptic plasticity model.

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Definition/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaM</td>
<td>calmodulin</td>
</tr>
<tr>
<td></td>
<td>UniProt:P62158</td>
</tr>
<tr>
<td>PP2B</td>
<td>calcineurin/protein phosphatase 2B</td>
</tr>
<tr>
<td></td>
<td>GO:0005955</td>
</tr>
<tr>
<td>CaMKII</td>
<td>calcium/calmodulin-dependent protein kinase II</td>
</tr>
<tr>
<td></td>
<td>GO:0005954</td>
</tr>
<tr>
<td>DARPP-32</td>
<td>dopamine- and cAMP-regulated phosphoprotein of 32 kDa</td>
</tr>
</tbody>
</table>
2.3 Calcium controls the relative activation

Table 2: continued

<table>
<thead>
<tr>
<th>Protein</th>
<th>Description</th>
<th>UniProt:Q9UD71</th>
<th>GO:0005952</th>
<th>GO:0000164</th>
</tr>
</thead>
<tbody>
<tr>
<td>PKA</td>
<td>cyclic AMP-dependent protein kinase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP1</td>
<td>protein phosphatase 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CBP</td>
<td>calcium binding protein</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: List of abbreviations used in the short-term synaptic plasticity model.

2.3.2 Calcium spikes and simulation design

Calcium signals in the spine are crucial for synaptic plasticity. Their transient changes are shaped by many factors including the calcium sources, calcium extrusion mechanisms, and the distribution of calcium buffer proteins. In this study, I focused on the calcium spikes induced by synaptic stimulation. Using the model described in the methods section, I showed that a single calcium input of 43200 molecules induced free intracellular calcium transient to reach the peak level, 1 micromolar (corresponding to 602 molecules), within 10 milliseconds, and decayed to basal level within 220 milliseconds (Fig. 10), which is in agreement with the amplitude and time course of NMDA receptor mediated calcium transient in an individual spine in partial depolarised conditions (Sabatini et al., 2002). This single input was repeated to induce a train of calcium spikes, with varied intervals, to form signals with different frequencies.

First, I modulated the calcium signal purely on frequency, by applying the same number of inputs, with the same input size. This generated a prolonged low-frequency stimulation, while a higher-frequency stimulus was relatively short-lived. In total, 34 different frequencies of calcium inputs, ranging from 0.1 Hz to 200 Hz, were studied in these simulations. For each frequency, 100 calcium inputs were applied after the system reached steady state (800 seconds after initiation, labelled as 0 second in the following figures). Therefore,
Figure 10: **Intracellular free calcium concentration increase induced by a single calcium input.** Increase of postsynaptic free calcium concentration triggered by a single calcium input. With a single input (43200 molecules), intracellular free calcium reaches maximal level, 1 micromolar (corresponding to 602 molecules), within 10 milliseconds, and decays back to basal level within 200 milliseconds.
while the simulations used different frequencies, equivalent amounts of calcium ions were used.

A train of inputs at 1 Hz did not change the peak level of intracellular free calcium (Fig. 11), when compared with a single input (Fig. 10). In contrast, a succession of inputs at 10 Hz and 25 Hz gradually raised not only the peak free intracellular calcium level but also the basal value (Fig. 11).

![Figure 11](image)

**Figure 11**: Intracellular free calcium concentration increase induced by a train of calcium inputs. Increase of postsynaptic free calcium concentration triggered by a train of inputs at 1 Hz (red line), 10 Hz (blue line), and 25 Hz (grey line). As the frequency of calcium inputs increases, the free calcium concentration rises both in its basal level and its maximal value.

Finally, the total number of calcium ions entering the spine was alternated in two ways. Firstly, the number of inputs was varied from 1 to 180, while the input size remained the same. Secondly, the input size was changed while the spike number was kept identical.

2.3.3 Parameter definition: the activated area

The activation of calcineurin and CaMKII by calmodulin displays distinct temporal characteristics. I integrated the concentration over time
of activated calcineurin and CaMKII, and calculated the area above the basal level of activity. This is defined as the ‘activated area’ (Fig. 12). This area represents both the duration and the amplitude of a stimulated enzyme and, since the catalytic activity is a constant, this effectively reflects the amount of substrate which will be affected (Le Novère et al., 2008). The ratio between the activated area of calcineurin and that of CaMKII was computed as a parameter to judge the preferential activation on calcineurin and CaMKII, after calcium stimulation at that specific frequency.

![Graph showing the definition of activated area](image)

Figure 12: **The definition of ‘activated area’**. The activated areas are highlighted. The time courses represent active calcineurin and CaMKII upon stimulation by a train of calcium spikes at 1 Hz. In this case, the activated area for calcineurin is 28.36, while that for CaMKII is 7.55.

### 2.3.4 Frequency-regulated activation of calmodulin

The frequency of calcium inputs plays a crucial role in the activation of calmodulin. The time course of calmodulin activation changed as a function of equal amplitude calcium inputs, but applied at varying frequencies (Fig. 13). In basal conditions, less than 10% of the total calmodulin was activated, and calcium inputs at low frequencies in-
duced no significant change in the activation of calmodulin. However, as the frequency increased, more calmodulin became active and the activation period was prolonged. At 50 Hz, almost 100% of the total calmodulin was in the active state, and more than half of the total calmodulin remained active for longer than 1 min. As a result, higher frequencies had no further effect. Therefore, the frequency of calcium inputs determined and stabilised the active state of calmodulin. At frequencies below 50 Hz, calmodulin was able to decode the calcium input frequency, and translated this information into the amount of active calmodulin, which was then transmitted to its targets.

Figure 13: Effects of calcium input frequencies on activation of calmodulin. Dependence of calmodulin activation on calcium input frequencies. Each curve represents a time course of normalised active calmodulin, stimulated by a train of calcium inputs at a specific frequency. Although the frequency differs between curves, the number of calcium inputs and the input size remain the same (100 inputs, 43200 molecules for each input).

2.3.5 Frequency-regulated calmodulin binding

The frequency of calcium spikes defined the ratio of calmodulin bound to calcineurin versus that bound to CaMKII. As the frequency in-
increased, more calmodulin bound to CaMKII rather than to calcineurin, and for increasing periods (Fig. 14).

![Diagram](image.png)

**Figure 14:** The ratio of calmodulin bound to calcineurin versus that bound to CaMKII. Each curve represents a time course of the ratio between calmodulin bound to calcineurin and that bound to CaMKII, for a train of calcium inputs at a specific frequency. For each frequency, the stimulus is composed of the same number of inputs with the same input size (100 inputs, 43200 molecules for each input).

As indicated above, calcium inputs at high frequencies steadily increased the activity of calmodulin. Because of the higher affinity of calmodulin for calcineurin compared to CaMKII, one could argue that calmodulin first binds to all the calcineurin molecules, then the additional calmodulin activates CaMKII. However, this was not the case because at the lowest ratio (about 0.02), approximately 34% of the total calcineurin was bound by calmodulin, which was far from being saturated. At the peak of calmodulin activation, about 39% of the total CaMKII was bound by calmodulin. Since the respective concentrations of CaMKII and calmodulin are 70 µM and 30 µM, this indicates the almost all the calmodulin molecules bound to CaMKII.
2.3.6 Frequency-modulated activation of calcineurin and CaMKII

The relative activation of calcineurin and CaMKII dictate the direction of synaptic plasticity, since their activation can induce opposing consequences on the synaptic strength. Fig. 15 and Fig. 16 show the time courses of calcineurin, CaMKII and PP1 in response to calcium inputs at two specific frequencies: 1 Hz and 50 Hz. In basal conditions, approximately 32% of the total calcineurin was active, and about 3% of the total CaMKII subunits were stimulated. This was due to the fact that less than 10% of the total calmodulin was active, and it tightly bound to calcineurin. Comparing to 1 Hz calcium stimulation, calcium input at 50 Hz triggered more significant activation of CaMKII, despite the pronounced activation of PP1 by calcineurin. At a 50 Hz frequency of calcium stimulation, calcineurin initially displayed a sharp activation, then plunged down to basal levels, prior to a second increase. This indicates that CaMKII competes with calcineurin for active calmodulin. However, high frequency calcium inputs did not prevent the activation of calcineurin. Instead, it shifted the balance from active calcineurin to active CaMKII. Thus, the information carried by calcium frequency was translated into the relative amplitude and duration of activation between these two proteins. As shown in Fig. 17, the ratio of active calcineurin versus CaMKII decreased when the frequency increased, indicating more CaMKII was active compared to calcineurin.

The information presented above is summarised in Fig. 18 (red line), which represents the ratio of calcineurin to CaMKII activated area (the calculation of ‘activated area’ is explained in section 2.3.3), in response to calcium inputs at 34 different frequencies. This suggests a frequency-dependent response. When calcium inputs were infrequent (e.g. 0.1 Hz), calcineurin activity was higher than that of CaMKII. However, an increase in frequency favoured a higher relative increase in CaMKII. In the 0.3-2.5 Hz input frequency range, there was a sharp drop in the ratio of stimulated calcineurin to CaMKII, indicating increased CaMKII activation. This shift of activity slowed down when the frequency further increased, and stabilised at 50 Hz, where the CaMKII activated area was more than twice that of calcineurin. Therefore, it was the frequency of calcium inputs that de-
2.3 Calcium controls the relative activation

![Graph showing the effects of low-frequency and high-frequency calcium signals on calciumMKII, calcineurin, and PP1 activation.](image)

**Figure 15:** Effects of a low-frequency calcium signal on activation of calcineurin and CaMKII. Time courses of normalised activated CaMKII, calcineurin, and PP1 in response to a 100-input calcium stimulation at 1 Hz.

![Graph showing the effects of high-frequency calcium signal on calcineurin and CaMKII activation.](image)

**Figure 16:** Effects of a high-frequency calcium signal on activation of calcineurin and CaMKII. Time courses of normalised activated CaMKII, calcineurin, and PP1 in response to a 100-input calcium stimulation at 50 Hz.
Figure 17: **Comparison of calcineurin and CaMKII activation induced by calcium inputs at different frequencies.** Dependence of the respective activation of calcineurin versus CaMKII on the frequency of calcium inputs. Each curve represents a time course of the ratio of activated calcineurin versus CaMKII, for a train of calcium inputs at a specific frequency. For each frequency, the stimulus is composed of the same number of inputs with the same input size (100 inputs, 43200 molecules for each input).
terminated the relative activation of calcineurin and CaMKII, with high calcium input frequency favouring CaMKII activation.

### 2.3.7 CaMKII autophosphorylation, a calcium input frequency decoder

As presented above, CaMKII was significantly stimulated when the frequency of calcium inputs increased above a threshold. It has been shown that CaMKII autophosphorylation on Thr286 results in a bistability of the enzyme activity (Zhabotinsky, 2000). Moreover, Thr286 phosphorylation is one mechanism by which CaMKII decodes calcium spike frequency in vitro (De Koninck and Schulman, 1998). The CaMKII autophosphorylation is indeed a frequency detector (Fig. 18). When the phosphorylation was prevented in the model (in silico mutation by setting the autophosphorylation rate as zero), the activation of CaMKII at high frequencies became less pronounced than in the wild type, and it hardly overcame the activation of calcineurin. In addition, the differences in response between the mutated and wild-type CaMKII become wider with increasing frequency of calcium spikes. In addition, when the frequency of calcium inputs became higher, the divergence of responses between the mutated and wild-type CaMKII became wider, which shows that the autophosphorylation of CaMKII influences mostly its response to calcium inputs at higher frequencies. This can also be interpreted as CaMKII autophosphorylation being mainly triggered at higher frequencies of calcium inputs.

The autophosphorylation of a CaMKII subunit on Thr286 requires that both this subunit and one adjacent neighbour are simultaneously active (Reviewed in Rosenberg et al., 2005). This requirement is the foundation of frequency-dependent autophosphorylation, since it establishes a threshold for activated calmodulin, beyond which CaMKII can be highly activated. Assuming there are only a few active calmodulin molecules able to bind CaMKII subunits, the low probability of two calmodulin molecules binding two adjacent subunits means very few subunits could be phosphorylated. Moreover, the phosphatase, which possesses a higher affinity for calmodulin, prevents the build up of this phosphorylation.

The autophosphorylation of CaMKII also causes calmodulin trapping. Autophosphorylation on Thr286 markedly increases the affin-
2.3 Calcium controls the relative activation

Figure 18: Effect of CaMKII autophosphorylation on the relative activation of calcineurin and CaMKII. Ratio of calcineurin to CaMKII activated area for the wild type (red line) and a CaMKII autophosphorylation mutant (at residue Thr286) (blue line). Each point represents the ratio of activated areas, upon the stimulation by a train of calcium inputs at a specific frequency (for the detailed calculation of “activated area”, see the methods section). Stimulation at each frequency is composed of 100 calcium inputs with the same input size (43200 molecules).
ity between CaMKII and calmodulin (Meyer et al., 1992), and locks CaMKII in the active state long after the end of calcium elevation. Therefore, the autophosphorylation facilitates CaMKII in the competition with other calmodulin-binding proteins. Thus, CaMKII responds to higher frequency signals with a positive feedback. This is clearly shown in the simulations where mutated CaMKII attracted less calmodulin during high-frequency stimulation than wild-type CaMKII.

2.3.8 Total amount of calcium ions

If the calcium input frequency is crucial for the activation of calcineurin and CaMKII, would there be a role for the absolute amount of calcium ions? To answer this question, I initially varied the total amount of calcium input by changing the number of inputs used in a stimulus, while maintaining the input amplitude as previously described (2.3.2). At a given frequency, CaMKII activation was proportional to the number of inputs (Fig. 19). Moreover, increased number of inputs resulted in increased CaMKII activity at low-frequency stimuli. However, when the number of inputs reached a certain threshold level, the increased quantity of calcium ions cannot further increase CaMKII sensitivity, even at lower-frequency stimuli. This ruled out a limiting role of calmodulin amount because calmodulin activation hardly saturated at low input frequencies (Fig. 13). Irrespective of the number of calcium inputs, the ratio of active calcineurin versus CaMKII reached the same level, indicating that the final extent of CaMKII activation at high calcium input frequencies was independent from the number of calcium inputs.

As discussed by Sabatini et al. (2002), the amplitude of the NMDA receptor-dependent calcium spike depends on the depolarization of the postsynaptic membrane, the peak concentration ranging from 0.7 to 12 µM. A second approach to vary the total calcium input is therefore to increase the size of each input while keeping the same number of inputs (100 inputs). For instance, the free intracellular calcium concentration reached 12 micromolar (corresponding to 7224 molecules) within 10 milliseconds, and declined over approximately 200 milliseconds, which is in agreement with experimental measurements (Fig. 20) (Sabatini et al., 2002). Akin to the increase of number in calcium
2.3 Calcium controls the relative activation

Figure 19: Effect of calcium input number on the relative activation of calcineurin and CaMKII. Comparison of different numbers of calcium inputs. Each point represents the ratio of calcineurin versus CaMKII activated area, when stimulated by a train of calcium inputs at a given frequency. The curves represent different numbers of calcium inputs, the amount for each input being the same (43200 molecules).
inputs, large calcium input sizes increased the proportion of active CaMKII at low frequencies, even when the frequency was as low as 0.1 Hz (Fig. 21). Once again, the final ratio of calcineurin versus CaMKII activity remained the same, indicating that the level of activated CaMKII at high-frequency stimulation is independent of the calcium input size as well. However, simulations with larger calcium input sizes reached this final ratio more rapidly than those with smaller sizes. Sabatini et al. (2002) argued that NMDA receptor-induced calcium influx at resting membrane potential triggers LTD, but induces LTP when coupled with postsynaptic membrane depolarization because of the large amplitude of calcium input. Combined with my simulation results, a more complete picture might be that large amplitude of calcium influx more easily induces LTP because it relies less on high spiking frequencies.

![Figure 20: Intracellular free calcium concentration increase induced by a large calcium input.](image)

Thus, high frequencies of calcium inputs provide a mechanism to produce transient but potent calcium elevations, in order to activate CaMKII. This is important when each calcium input is not
2.3 Calcium controls the relative activation

Figure 21: Effect of calcium input size on the relative activation of calcineurin and CaMKII. Comparison of the relative activation between calcineurin and CaMKII, stimulated by different sizes of calcium input. Each curve represents a different size of calcium input, while the labelled concentration indicates the peak level a free calcium spike reaches after this input. Each point represents the ratio of calcineurin versus CaMKII activated area, when stimulated by a train of calcium inputs at a given frequency.
large enough, or the number of calcium inputs is limited. In parallel, a large amount of calcium ions entering into the spine can lower the threshold frequency required for activating CaMKII. Therefore, the total amount of inputted calcium ions contributes to the sensitivity of the system towards decoding the frequencies, and determines above which frequency CaMKII can overcome calcineurin, and when LTP can be triggered over LTD. However, the final extent to which CaMKII can be activated seems independent from the actual quantity of calcium ions, though it may depend on other factors, such as phosphatase inhibitors, and the availability of calmodulin.

2.3.9 Calmodulin availability

Calmodulin not only activates CaMKII, but its availability also influences how CaMKII responds to calcium input frequency (Klee, 1991; Luby-Phelps et al., 1995). When calmodulin concentration was increased from 30 micromolar to 45 micromolar and 60 micromolar, the amount of CaMKII activation at low frequencies of calcium stimulation increased (Fig. 22). In addition, the stabilised CaMKII activation at high calcium frequencies rose when the total calmodulin concentration increased. It therefore seems that calmodulin plays a dual role in the activation of CaMKII. Besides its ability to activate CaMKII subunits, the amount of calmodulin also limits the extent of CaMKII autophosphorylation on Thr286, in response to different frequencies. As free calmodulin is less abundant than CaMKII in the spine (Xia and Storm, 2005), the availability of calmodulin limits the chance of a CaMKII subunit having a neighbouring subunit in the active state and able to phosphorylate it. If we view autophosphorylation and calmodulin trapping as a cooperative process, calmodulin availability would determine the input frequencies that induce the onset of this process. Besides, there may be active recruitment processes for calmodulin taking place in specific locations within a spine, for instance, in the area near calcium channels in the PSD. According to our simulation results, in these regions, CaMKII activation can be induced by much lower frequencies of calcium influx, and may play an important role in modulating synaptic plasticity.
2.3 Calcium controls the relative activation

Figure 22: Effect of calmodulin concentration on the relative activation of calcineurin and CaMKII. Comparison of the relative activation between calcineurin and CaMKII, when different concentrations of calmodulin are available. Calmodulin concentration varies among different curves, but the number of calcium inputs and the amount of molecules for each input remain the same (100 inputs, 43200 molecules per input). On a given curve, each point shows the ratio of activated area of calcineurin versus CaMKII, when stimulated by a train of calcium inputs at a given frequency.
2.3 Calcium controls the relative activation

2.3.10 PP1 inhibition by DARPP-32

DARPP-32 interacts with PP1 through two binding domains, which contribute to both low affinity binding and inhibition (Hemmings et al., 1990; Desdouits et al., 1995a; Kwon et al., 1997; Huang et al., 1999). Upon binding both sites, the affinity between DARPP-32 and PP1 is increased to a nanomolar range (Desdouits et al., 1995a; Hemmings et al., 1984a). In my model, a relatively low affinity ($K_d=100$ nM) was used, representing the average inhibitory effect of different PP1 inhibitors, under basal conditions. However, when the affinity between DARPP-32 and PP1 was increased to approximately 70-fold ($K_d=1.5$ nM) (Hemmings et al., 1984a), with basal DARPP-32 phosphorylation unchanged, the activation of CaMKII effectively rose during low-frequency calcium stimulation (Fig. 23). Moreover, the size of CaMKII activated area was 10 times that of calcineurin in the high-frequency range (50 Hz to 200 Hz), which was a 5-fold increase when compared with the ratio obtained in the low affinity simulation (Fig. 23). This indicates that, although CaMKII activation outpaces its phosphatase at high frequency stimulation (Fig. 16), PP1 still plays an important role in shaping CaMKII response to calcium. The potency of PP1, which is controlled by DARPP-32, not only determines the sensitivity of the CaMKII frequency response, but also the extent to which CaMKII can be activated by high frequency calcium input. This points to other avenues of controlling synaptic plasticity via other signalling pathways.
2.3 Calcium controls the relative activation

Figure 23: Effect of increased PP1 inhibition on the relative activation of calcineurin and CaMKII. Comparison of the relative activation of calcineurin and CaMKII, between different inhibitory potentials of DARPP-32 on PP1. The constant for competitive inhibition changes between the curves, but the number of calcium inputs and the amount of molecules for each input remain the same (100 inputs, 43200 molecules per input). On a given curve, each point shows the ratio of relative activation between calcineurin and CaMKII, stimulated by a train of calcium inputs at a specific frequency.
2.4 Discussion

The influx of calcium through the NMDA receptor is of particular importance for synaptic plasticity. However, the mechanisms underlying the dual role of calcium, triggering either LTP or LTD, are still largely unclear. In the present study, I explored how postsynaptic calcium elevation could drive different biochemical cascades by inducing different responses of calcineurin and CaMKII according to calcium input frequency, amplitude, and duration. I showed that increased calcium input frequency increases both the activation of calcineurin and CaMKII. However, changing input frequency shifts the balance of activation between the two enzymes. In addition, the activation of CaMKII can be achieved either by high-frequency calcium inputs, or at low frequencies by stimulation with large inputs.

My simulation results are in agreement with previous experimental reports. Low-frequency calcium input lasting longer induced large activation of calcineurin, demonstrated as follows. First, Mulkey and Malenka (1992) showed that the electrical induction of LTD resulting from low-frequency afferent stimulation needs to last for 30 s. Second, Yang et al. (1999) showed that a moderate increase in calcium for about 1 min can induce LTD. Short high frequency calcium inputs preferentially triggered the activation of CaMKII. This is also supported by experimental evidence showing that LTP is elicited by stimulation transiently raising calcium to significantly high levels (Petrozzino et al., 1995) for about 2 seconds (Malenka et al., 1992; Yang et al., 1999). In my simulations, larger calcium input sizes allowed CaMKII activation at lower calcium input frequencies (Fig. 21). This has been demonstrated in experiments showing that paired low-frequency afferent stimulation with postsynaptic depolarization results in LTP rather than LTD (Gustafsson et al., 1987). In addition, this computational model provided useful information for the range of frequencies of postsynaptic calcium spikes, which could induce LTP.

High-frequency stimulation induces short but potent elevation of the free intracellular calcium concentration, and these specific calcium transients place a specific temporal constraint for calcium decoding proteins, such as calmodulin. I demonstrated that calmodulin
activation depends on the calcium firing frequencies. With a constant total amount of calcium ions input, high frequency inputs stimulate calmodulin more efficiently.

Another decoder of calcium input frequency is CaMKII, due to its multimeric structure and intersubunit autophosphorylation. As introduced above, high-frequency calcium inputs induce the activation of calmodulin, which enhances its probability of binding to neighbouring subunits, thus inducing CaMKII autophosphorylation. Most importantly, this autophosphorylation promotes high affinity calmodulin binding (Meyer et al., 1992; Hanson et al., 1994) and converts this kinase into Ca\(^{2+}\)/calmodulin independent state (Miller and Kennedy, 1986; Mukherji and Soderling, 1994). CaMKII frequency decoding ability has been demonstrated in vitro. De Koninck and Schulman (1998) showed that CaMKII autophosphorylation occurs in a frequency-dependent manner and this frequency response is modulated by the amplitude and duration of each calcium pulse, which is congruent with my findings. Although the calcium input frequency shifts the balance of activated calcineurin and CaMKII, the total amount of calcium ions in the signal defines how sensitive the system is towards different calcium input frequencies. In other words, large amounts of calcium ions lower the frequency threshold that induces the autophosphorylation of CaMKII (Fig. 19 and Fig. 21). Thus, variation of calcium input frequency may be a strategy for neurons to build up transient calcium concentration when total calcium input is limited. The activation of CaMKII can equally be induced either by high frequency of calcium spikes or a large amount of calcium ions.

Interestingly, the range of frequencies sensitive to the activation of signal goes from 3 to 50 Hz, corresponding roughly to the frequencies used to induce postsynaptic LTD and LTP (Mulkey and Malenka, 1992; Petrozzino et al., 1995).

LTP and LTD are two widespread phenomena regulating the strength of possibly every excitatory synapse. They demonstrate how each synapse can undergo long-lasting modifications in response to synaptic activity. There are many different forms of LTP and LTD, and their underlying molecular mechanisms vary depending on the synapses and the neuronal circuits involved (Malenka and Bear, 2004). In the MSN of the striatum, dopamine afferents modulate glutamate-induced syn-
aptic plasticity through DARPP-32 inhibition of PP1, thus regulating CaMKII, and the neuronal plasticity at the heart of drug addiction (Anderson et al., 2008). DARPP-32 is highly expressed in MSN, and its high reported affinity binding to PP1 means that inhibition is almost irreversible. As shown above (Fig. 23), this inhibition increases CaMKII sensitivity towards low frequency calcium firing. Hence, in MSN, the activation of dopamine D1 receptor and the DARPP-32 pathway can trigger LTP, as shown by Wolf et al. (2004). Furthermore, repeated in vivo treatments with psychostimulants increases the surface expression of AMPA receptors in the striatum (Boudreau and Wolf, 2005). However, paradoxically, a single cocaine injection in drug-naive animals exerts no effect on synaptic plasticity, while in drug-experienced animal it induces LTD (Kourrich et al., 2007). It seems that, although drugs of abuse could theoretically influence short-term synaptic plasticity and induce potentiation, these psychostimulants exert their effects through a more complicated procedure involving long-term neuroadaptation. Nevertheless, the computational model presented here improved our understanding of calcium signalling involved in synaptic plasticity. The frequency of postsynaptic calcium influx regulates the induction of LTP and LTD, while the amount of calcium ions shifts the windows of frequencies required for this bidirectional regulation. Furthermore, synaptic plasticity is induced in a cell-specific manner, and is modulated by other pathways, such as the dopamine regulated PP1 inhibition in MSN. Finally, frequency adds complexity to calcium signals, thereby increasing the spectrum and accuracy of control on downstream targets.
3.1 Introduction

Long-term neuroadaptation relies largely on morphological alterations of neurons and modifications of synaptic connections, the so-called structural plasticity (Brown and Kolb, 2001; Robinson and Kolb, 1999a). Some of the processes regulating structural plasticity are the same as those for the early phase synaptic plasticity. NMDA receptor activation leads to a postsynaptic calcium elevation, which activates kinases and GTPases, and modulates glutamate receptors. Among these calcium effectors, accumulating evidence indicates that MAP kinases and, in particular, ERK1/2, are crucial in regulating structural changes such as dendritic remodelling and the density of synaptic connections (Wu et al., 2001b; Thomas and Huganir, 2004).

ERK1/2, their upstream regulators, and their downstream targets, have been found to be highly expressed in neurons (Boulton et al., 1991). ERK1/2 can directly modulate the transport of AMPA receptors to the postsynaptic membrane (Zhu et al., 2002), a key process in inducing LTP (Hayashi et al., 2000; Lu et al., 2001; Shi et al., 2001), and in stabilising rapid morphology changes of the spine (Fischer et al., 2000). Sustained activation of ERK1/2 is essential for new spine formation in hippocampal neurons (Wu et al., 2001b). Activated ERK1/2 can translocate to the nucleus, and regulate several key transcription factors crucial for long-term memory. This regulation may be direct, as in the case of Elk1 (Berman et al., 1998; Davis et al., 2000; Sananbenesi et al., 2002) or indirect, as for CREB, where it is mediated through control of the mitogen- and stress-activated kinases (MSK)s (Deak et al., 1998), and ribosomal protein S6 kinases (RSK)s (Xing et al., 1996).

In neuronal cells, ERK1/2 activation can be induced by glutamate signalling via elevated postsynaptic calcium level (Fiore et al., 1993; Zhu et al., 2002). Calcium influx activates Ras by controlling
the balance of activation between Ras guanine nucleotide exchange factor protein (Ras-GEF) and Ras GTPase activating protein (Ras-GAP) (Thomas and Huganir, 2004). Activated Ras leads to the activation of Raf, which in turn phosphorylates the dual-specificity MAPK/ERK kinase (MEK). ERK1/2 are unique MAP kinases, because their full activation requires the phosphorylation not only of a threonine (Thr183) but also of a tyrosine (Tyr185) residue by MEK (Anderson et al., 1990; Robinson and Cobb, 1997). The deactivation of ERK1/2 can be achieved by three groups of phosphatases: the dual specific phosphatases (Camps et al., 2000), the serine/threonine phosphatases (Alessi et al., 1995), and the tyrosine phosphatases (Pulido et al., 1998; Saxena et al., 1999) (PTPs). In the case of the serine/threonine phosphatases, ERK1/2 can be effectively dephosphorylated by PP2A but not by calcineurin or PP1 (Zhou et al., 2002).

In the striatum, the administration of various drugs of abuse activates extracellular signal regulated kinase-2 (ERK2) through the combined stimulation of the glutamate NMDA receptor and the dopamine D1 receptor (Valjent et al., 2000, 2005; Salzmann et al., 2003). The consequences of such activation are the increased intracellular levels of two major second messengers, Ca2+ and cAMP, respectively. Increased cAMP levels lead to the phosphorylation of DARPP-32 on Thr34; calcium elevation results in the activation of MAP kinase pathway, but also in the dephosphorylation of DARPP-32 on Thr34. Interestingly, acute cocaine injection to a mouse carrying either inactivated DARPP-32 or a Thr34Ala mutation results in attenuated ERK2 activation, indicating that cocaine effectively stimulates DARPP-32 phosphorylation on Thr34, and that this phosphorylation is specifically required for activation of ERK2. DARPP-32 is highly expressed in MSN and is a potent PP1 inhibitor once phosphorylated on Thr34 (Hemmings et al., 1990). Therefore, phospho-Thr34 DARPP-32 might enhance ERK1/2 activity by inhibiting PP1. Since PP1 cannot directly dephosphorylate ERK1/2(Zhou et al., 2002), how PP1 negatively regulates ERK1/2 is still an open question.

While the glutamate-dependent signalling pathways activate the phosphorylation of ERK1/2, it is likely that dopamine exerts its effect by inhibiting phosphatases acting on ERK1/2. The striatal-enriched tyrosine phosphatase (STEP) has been implicated as playing a key role...
role, linking DARPP-32 to the MAP kinase pathway (Valjent et al., 2005; Girault et al., 2007). It has also been established that N-methyl-D-aspartate receptor (NMDAR)-mediated calcium elevation activates STEP, leading to the dephosphorylation of ERK2 on the tyrosine residue (Paul et al., 2003). STEP loses its phosphatase activity when phosphorylated by PKA (Paul et al., 2000). Its dephosphorylation, which may involve several potential phosphatases, restores this phosphatase activity. One homolog of STEP, HePTP, can be activated by PP1, but not by calcineurin (Nika et al., 2004). Therefore, it is possible that dopamine positively modulates ERK1/2 activity through DARPP-32 inhibition on PP1, and the subsequent inactivation of STEP.

Conversely, Paul et al. (2003) raised the possibility that calcineurin, instead of PP1, activates STEP. Indeed, after exposure to a calcineurin inhibitor, cyclosporin A, glutamate stimulation results in higher levels of tyrosine-phosphorylated ERK2 than that are observed in the control. Further, this phosphorylation level persists 30 minutes post-stimulation. Moreover, glutamate fails to activate STEP when calcineurin is inhibited. It therefore seems that DARPP-32 has no influence on STEP activity. However, it is difficult to conclude that calcineurin activates STEP directly, because it can release PP1 inhibition by dephosphorylating DARPP-32 at Thr34, thus inhibiting calcineurin decreases PP1 activity as well.

NMDA receptors can trigger postsynaptic calcium influx, activating second messenger signalling pathways, and controlling activity-dependent synaptic plasticity. NMDA receptors are tetrameric complexes composed of two NR1 and two NR2 subunits (Sheng et al., 1994; Kim et al., 2005). The identities of NR2 subunits, especially the relative expression levels of NR2A and NR2B, vary according to neuronal differentiation stage. In mice, NR2B-containing NMDA receptors are strongly expressed during birth, and slowly decline during adulthood. NR2A-containing NMDA receptors appear around postnatal day 7 and gradually increase during adulthood (Monyer et al., 1994; Sheng et al., 1994; Barria and Malinow, 2005). NR2A- and NR2B-containing receptors not only have distinct gating kinetics (Cull-Candy et al., 2001), but also have opposing effects on regulating Ras-ERK1/2 signalling pathway. It has been shown that NR2B-containing NMDA receptors negatively regulate ERK1/2 activity, the
converse of the effect exerted by NR2A-containing receptors (Kim et al., 2005).

Synaptic Ras GTPase activating protein (SynGAP) is a major component of the PSD (Cheng et al., 2006). In cultured neurons, SynGAP inhibits ERK1/2 (Rumbaugh et al., 2006). SynGAP associates with NR2B via the binding of PSD95/SAP102’s PDZ domain, or via indirect connection with CaMKII (Kim et al., 1998; Vazquez et al., 2004; Krapivinsky et al., 2004). CaMKII translocates from cytosol to the PSD, and associates with NR2B following treatments that induce autophosphorylation of CaMKII and LTP (Strack et al., 1997b; Leonard et al., 1999; Strack et al., 2000; Meng and Zhang, 2002). Furthermore, Oh et al. (2004) identified four specific serine residues of SynGAP which, once phosphorylated by CaMKII, lead to increased RasGAP activity. Therefore, the negative regulation of NR2B-containing NMDA receptor on MAP kinase pathway may be due to the preferential coupling of SynGAP and activated CaMKII to NR2B (Kim et al., 2005). Thus, PP1 can inhibit SynGAP and exert positive regulation on ERK1/2, because it dephosphorylates CaMKII, preventing it from activating SynGAP. Therefore, PP1 may have dual roles in regulating ERK1/2 activity, and would shift between roles according to neuronal differentiation stage.

In the following sections I will first address the question of which phosphatase activates STEP, PP1 or calcineurin, by using computational modelling. I will then describe experimental results conducted on primary striatal neuron cultures revealing a peculiar inhibitory effect of glutamate on ERK activity. Finally, I will present a detailed computational model that provides a tentative explanation of the opposite effects displayed by PP1 on ERK activity according to different neuronal differentiation, and in particular the type of NMDA receptors expressed.

### 3.2 Experimental methods

The methods used to model the signalling pathways and run numerical simulations are presented in 2.2. In addition to Ca\(^{2+}\) perturbation, the simulation of MAP kinase model also requires the cAMP input,
which was realised by increasing the cAMP influx constant for 5 milliseconds for each spike.

Phosphatase inhibition \textit{in silico} was achieved by increasing the concentration of phosphatase inhibitor, which processes high affinity binding to phosphatase (as listed in Table 3), 50 min prior to calcium input. Specifically, 5 µM tautomycetin was used for inhibiting PP1, while 5 µM cyclosporin A for inhibiting calcineurin.

\subsection*{3.2.1 Cell culture and glutamate stimulation}

Primary neuronal cultures were obtained from 15-day embryonic Swiss mice. Striatum were dissected under the microscope, and the retrieved tissues was incubated with trypsin (0.001 g/mL) under 37°C water bath for 30 min. The trypsinized striatum was then quenched using DNase (0.004 g/mL) and heat-inactivated fetal bovine serum (10%) and centrifuged (900 rpm, 5 min, room temperature). The supernatant was removed, and the tissue pellet was triturated in neurobasal media. Cells were plated on tissue culture dishes (800 000 cells per 35 mm culture dish for western blot; 150 000 cells per 14 mm culture dish for immunofluorescence) for 7 or 14 days at 37°C in a humidified atmosphere consisting of 5% CO2. Culture dishes had been coated with Poly-L-lysine (50 µg/mL) in an incubator at 37°C overnight, prior to cell plating.

A protein phosphatase inhibitor, either tautomycetin (5 µM for inhibiting PP1) or okadaic acid (500 nM for inhibiting PP2A) was added into the cell culture, and incubated at 37°C for 50 min prior to glutamate stimulation (using 50 µM). Incubation of the stimulated cultures was terminated at various time intervals, as indicated in the results. Control groups were treated with DMSO, instead of tautomycetin or okadaic acid, in equal concentration and incubated for equal time intervals.

Neuronal cultures grown on coverslips for immunofluorescence received the same treatment of protein phosphatase inhibitors, as indicated above. However, they were only treated with glutamate for 0 or 10 minutes before fixation.
3.2.2 Western blotting

After treatment by phosphatase inhibitors and glutamate stimulation, the cells were washed twice with ice-cold phosphate-buffered saline (PBS), and were lysed in sodium dodecyl sulphate (SDS) (1%) and sodium orthovanadate (Na$_3$VO$_4$) (1 mM), followed by sonication (2x33 s, 43 W), then heated for 5 minutes (100°C).

Each sample (containing 30 µg protein) was separated by 10% SDS-polyacrylamide gel electrophoresis (SDS-PAGE) followed by western blotting. Activated ERK1/2 and total ERK1/2 were detected by probing the blots with mouse monoclonal antibodies against diphospho-ERK1/2 (1:1000, Sigma) and ERK1/2 (1:5000, Sigma).

3.2.3 Immunofluorescence of striatal neurons in culture

After treatment by phosphatase inhibitors and glutamate stimulation, the cells were washed twice with PBS, then fixed with PBS containing 2% paraformaldehyde (40 min, room temperature). After three rinses in PBS, cells were treated with 0.1% Triton for 15 minutes, then washed three rinses with PBS. Later, cells were treated with blocking buffer (3% Bovine Serum Albumin (BSA) in PBS) for 1 hour at room temperature, then incubated overnight at 4°C in 1% BSA with diphospho-ERK1/2 (1:1000 Cell signalling, rabbit polyclonal) and MAP2 (1:1000 Signal, mouse monoclonal) antibodies. After three rinses with PBS, anti-mouse Alexa488-conjugated antibody and anti-mouse Cy3-conjugated antibody (1:400 Molecular Probes) were added, and cells incubated for 1 h at room temperature.

Cells were rinsed three times with PBS prior to mounting under coverslips with Vectashield with 4′,6-diamidino-2-phenylindole (DAPI) labelling nucleus (Vector Laboratories).

3.3 Phosphatase regulating STEP activity

In order to decipher which phosphatase stimulates STEP, calcineurin or PP1, I compared the results obtained from two computational models with existing experimental measurements.
Model structure and simulation plan

The two models described here (Fig. 24), share the majority of interactions, including the dopamine activated DARPP-32 pathway and glutamate stimulated MAP kinase pathway. The only difference between the models is the activation of STEP, either by calcineurin or by PP1. All the quantitative parameters used in these two models are listed in Table 3, and the abbreviations are explained in Table 4. The models and the Python script for simulation in E-Cell, are available online (http://www.ebi.ac.uk/~luli/thesis/mapk/step/).
Computational models of the regulation of STEP activity. The graphical conventions are those of the Process Description language of Systems Biology Graphical Notation (SBGN) (Le Novère et al., 2009). Rectangular box with rounded corners: macromolecule, circular container: simple chemical, elliptical container: state variable, "empty set": a source or sink, square on the arc: process, square containing parallel slanted lines: omitted process, square containing question mark: uncertain process, filled disc on the arc: association, filled arrow head: production, empty arrow head: stimulation, empty circle: catalysis.
Table 3: List of parameters used for simulation of the MAP kinase model that concerns PP1 or calcineurin (PP2B) activates STEP.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ca^{2+} ) pump:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>( 4 \times 10^{-3} \text{ Ms}^{-1} )</td>
<td>(cf. Markram et al., 1998)*,[1]</td>
</tr>
<tr>
<td>( K_m )</td>
<td>( 1 \times 10^{-6} \text{ M} )</td>
<td>(Markram et al., 1998)*</td>
</tr>
<tr>
<td>( Ca^{2+} ) leak:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>( 3 \times 10^{-4} \text{ Ms}^{-1} )</td>
<td>(cf. Markram et al., 1998)*,[1]</td>
</tr>
<tr>
<td>( Ca^{2+} ) binding calcium buffer protein (CBP):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{\text{on}}_{\text{CBPfast}} )</td>
<td>( 10^9 \text{ M}^{-1} \text{s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{off}}_{\text{CBPfast}} )</td>
<td>( 10^3 \text{ s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{on}}_{\text{CBPmedium}} )</td>
<td>( 10^8 \text{ M}^{-1} \text{s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{off}}_{\text{CBPmedium}} )</td>
<td>( 10^2 \text{ s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{on}}_{\text{CBPslow}} )</td>
<td>( 10^7 \text{ M}^{-1} \text{s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{off}}_{\text{CBPslow}} )</td>
<td>( 10 \text{ s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{on}}_{\text{CBPvslow}} )</td>
<td>( 10^6 \text{ M}^{-1} \text{s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( k_{\text{off}}_{\text{CBPvslow}} )</td>
<td>( 1 \text{ s}^{-1} )</td>
<td>(Markram et al., 1998)*,[2]</td>
</tr>
<tr>
<td>( CaM ) activation (Hill equation):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_d )</td>
<td>( 7 \times 10^{-6} \text{ M} )</td>
<td>(D’Alcantara et al., 2003)*,[3]</td>
</tr>
<tr>
<td>( n_H )</td>
<td>( 4 )</td>
<td>(D’Alcantara et al., 2003)*,[3]</td>
</tr>
</tbody>
</table>
Table 3: continued

Phosphatase regulating STEP activity

CaM inactivation:

\[ k = 2.5 \text{s}^{-1} \]

this study*

PP2B binding Ca\(^{2+}\) (Hill equation):

\[ K_d = 2 \times 10^{-7} \text{M} \]

this study*

\[ n_H = 1.6 \]

this study*

Ca\(^{2+}\) ions dissociate from PP2B:

\[ k = 5 \text{s}^{-1} \]

this study*

PP2BCa4 binding CaM Ca4:

\[ k_{on_{PP2B}} = 6 \times 10^9 \text{M}^{-1}\text{s}^{-1} \]

(Meyer et al., 1992)\(^{[4]}\)

\[ k_{off_{PP2B}} = 6 \text{s}^{-1} \]

(Meyer et al., 1992)\(^{[4]}\)

cAMP in (constant flux):

\[ k = 2.3 \times 10^{-7} \text{Ms}^{-1} \]

(estimated from Bacskai et al., 1993)*\(^{[5]}\)

cAMP pump:

\[ k = 4 \text{s}^{-1} \]

(estimated from Bacskai et al., 1993)*\(^{[5]}\)

PKA activation:

\[ k_{on_{R2C2_{cAMP}}} = 5.4 \times 10^7 \text{M}^{-1}\text{s}^{-1} \]

(Bhalla and Iyengar, 1999)*\(^{[6]}\)

\[ k_{off_{R2C2_{cAMP}}} = 33 \text{s}^{-1} \]

(Bhalla and Iyengar, 1999)*\(^{[6]}\)

\[ k_{on_{AMP_R2C2_{cAMP}}} = 5.4 \times 10^7 \text{M}^{-1}\text{s}^{-1} \]

(Bhalla and Iyengar, 1999)*\(^{[6]}\)

\[ k_{off_{AMP_R2C2_{cAMP}}} = 33 \text{s}^{-1} \]

(Bhalla and Iyengar, 1999)*\(^{[6]}\)

\[ k_{on_{AMP2R2C2_{cAMP}}} = 7.5 \times 10^7 \text{M}^{-1}\text{s}^{-1} \]

(Bhalla and Iyengar, 1999)*\(^{[7]}\)
### Table 3: continued

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Rate Constant</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{off, AMPR2C2_AMP}$</td>
<td>$10^2$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*{[7]}$</td>
</tr>
<tr>
<td>$k_{on, AMPR2C2_AMP}$</td>
<td>$7.5 \times 10^7$ M$^{-1}$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*{[6]}$</td>
</tr>
<tr>
<td>$k_{off, AMPR2C_PKA}$</td>
<td>$10$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*$</td>
</tr>
<tr>
<td>$k_{on, AMPR2C_PKA}$</td>
<td>$1.8 \times 10^7$ M$^{-1}$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*$</td>
</tr>
<tr>
<td>$k_{off, AMPR2P_KA}$</td>
<td>$10$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*$</td>
</tr>
<tr>
<td>$k_{on, AMPR2P_KA}$</td>
<td>$1.8 \times 10^7$ M$^{-1}$ s$^{-1}$</td>
<td>(Bhalla and Iyengar, 1999)$^*$</td>
</tr>
</tbody>
</table>

**PKA phosphorylates PDE:**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Rate Constant</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$6 \times 10^6$ M$^{-1}$ s$^{-1}$</td>
<td>(Fernandez et al., 2006)$^*$</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$36$ s$^{-1}$</td>
<td>(Fernandez et al., 2006)$^*$</td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$9$ s$^{-1}$</td>
<td>(Fernandez et al., 2006)$^*$</td>
</tr>
</tbody>
</table>

**PDEp dephosphorylation:**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Rate Constant</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$0.1$ s$^{-1}$</td>
<td>(Fernandez et al., 2006)$^*$</td>
</tr>
</tbody>
</table>

**PKA phosphorylates PP2A:**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Rate Constant</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$2.4 \times 10^6$ M$^{-1}$ s$^{-1}$</td>
<td>(Usui et al., 1998)$^*[8]$</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$0.1$ s$^{-1}$</td>
<td>(Usui et al., 1998)$^*[8]$</td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$0.3$ s$^{-1}$</td>
<td>(Usui et al., 1998)$^*[8]$</td>
</tr>
</tbody>
</table>

**PP2Ap dephosphorylation:**
Table 3: continued

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.08 s$^{-1}$</td>
<td>in this study$^*$</td>
</tr>
</tbody>
</table>

**cAMP degradation:**

| $k_{on,cAMP,PDE}$ | $8 \times 10^7$ M$^{-1}$s$^{-1}$ | (Fernandez et al., 2006)$^*$ |
| $k_{off,cAMP,PDE}$ | 40 s$^{-1}$ | (Fernandez et al., 2006)$^*$ |
| $k_{cat,cAMP,PDE}$ | 10 s$^{-1}$ | (Fernandez et al., 2006)$^*$ |
| $k_{on,cAMP,PDE}$ | $8 \times 10^7$ M$^{-1}$s$^{-1}$ | (Fernandez et al., 2006)$^*$ |
| $k_{off,cAMP,PDE}$ | 80 s$^{-1}$ | (Fernandez et al., 2006)$^*$ |
| $k_{cat,cAMP,PDE}$ | 20 s$^{-1}$ | (Fernandez et al., 2006)$^*$ |

**PKA phosphorylates DARPP-32 on Thr34:**

| $k_{on,D,PKA}$ | $5.6 \times 10^6$ M$^{-1}$s$^{-1}$ | (Hemmings et al., 1984b)$^9$ |
| $k_{off,D,PKA}$ | 10.8 s$^{-1}$ | (Hemmings et al., 1984b)$^9$ |
| $k_{cat,D,PKA}$ | 2.7 s$^{-1}$ | (Hemmings et al., 1984b)$^9$ |
| $k_{on,Dp75,PKA}$ | $5.6 \times 10^6$ M$^{-1}$s$^{-1}$ | (estimated from Bibb et al., 1999)$^{10}$ |
| $k_{off,Dp75,PKA}$ | 0.3 s$^{-1}$ | (estimated from Bibb et al., 1999)$^{10}$ |
| $k_{cat,Dp75,PKA}$ | 0 s$^{-1}$ | (Bibb et al., 1999)$^{10}$ |

**PP2B$_{active}$ dephosphorylates DARPP-32 on Thr34:**

| $k_{on}$ | $4.1 \times 10^6$ M$^{-1}$s$^{-1}$ | (King et al., 1984)$^{11}$ |
Table 3: continued

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{off}})</td>
<td>(6.4 , \text{s}^{-1})</td>
<td>(King et al., 1984)(^{[11]})</td>
</tr>
<tr>
<td>(k_{\text{cat}})</td>
<td>(0.2 , \text{s}^{-1})</td>
<td>(King et al., 1984)(^{[11]})</td>
</tr>
</tbody>
</table>

CDK5 phosphorylates DARPP-32 on Thr75:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{on}})</td>
<td>(6.2 \times 10^6 , \text{M}^{-1}\text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1984(^{b}))(^{[9]})</td>
</tr>
<tr>
<td>(k_{\text{off}})</td>
<td>(12 , \text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1984(^{b}))(^{[9]})</td>
</tr>
<tr>
<td>(k_{\text{cat}})</td>
<td>(2.5 , \text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1984(^{b}))(^{[9]})</td>
</tr>
</tbody>
</table>

PP2A dephosphorylates DARPP-32 on Thr75:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{on}})</td>
<td>(2.74 \times 10^6 , \text{M}^{-1}\text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1990)(^{[12]})</td>
</tr>
<tr>
<td>(k_{\text{off}})</td>
<td>(116 , \text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1990)(^{[12]})</td>
</tr>
<tr>
<td>(k_{\text{cat}})</td>
<td>(10 , \text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1990)(^{[12]})</td>
</tr>
</tbody>
</table>

PP2Ap dephosphorylates DARPP-32 on Thr75:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{\text{on}})</td>
<td>(2.74 \times 10^6 , \text{M}^{-1}\text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1990)(^{[12]})</td>
</tr>
<tr>
<td>(k_{\text{off}})</td>
<td>(18.4 , \text{s}^{-1})</td>
<td>(estimate from Hemmings et al., 1990)(^{[12]})</td>
</tr>
</tbody>
</table>
Table 3: continued

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$k_{cat}$</th>
<th>$k_{on}$</th>
<th>$k_{off}$</th>
<th>$k_{cat}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phosphatase regulating STEP activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DARPP-32Thr34p binding PP1:</td>
<td>$20 \text{s}^{-1}$</td>
<td>$4.0 \times 10^6 \text{M}^{-1}\text{s}^{-1}$</td>
<td>$0.06 \text{s}^{-1}$</td>
<td></td>
</tr>
<tr>
<td><strong>PP1/PP2B dephosphorylates STEP:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on}$</td>
<td>$3 \times 10^6 \text{M}^{-1}\text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$32 \text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$0.2 \text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>PKA phosphorylates STEP:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on}$</td>
<td>$4.1 \times 10^6 \text{M}^{-1}\text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$10 \text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$0.67 \text{s}^{-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Raf activation by CaM:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| $k_{cat}$ | $10 \text{s}^{-1}$ | | | this study$^*$
| $K_m$ | $3 \times 10^{-7} \text{M}$ | | | this study$^*$
| **Raf inactivation:** | | | | |
| $v_{max}$ | $1 \times 10^{-6} \text{Ms}^{-1}$ | | | this study$^*$
### Table 3: continued

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$</td>
<td>$4.5 \times 10^{-5} M$</td>
<td>this study*</td>
</tr>
</tbody>
</table>

**Raf phosphorylates MEK or MEKp:**

| $k_{on}$ | $3.3 \times 10^6 M^{-1}s^{-1}$ | (Sasagawa et al., 2005)* |
| $k_{off}$ | $0.42 s^{-1}$ | (Sasagawa et al., 2005)* |
| $k_{cat}$ | $0.105 s^{-1}$ | (Sasagawa et al., 2005)* |

**PP2A or PP2Ap dephosphorylates MEKp or MEKpp:**

| $k_{on}$ | $8 \times 10^6 M^{-1}s^{-1}$ | (Sasagawa et al., 2005)* |
| $k_{off}$ | $122 s^{-1}$ | (Sasagawa et al., 2005)* |
| $k_{cat}$ | $3 s^{-1}$ | (Sasagawa et al., 2005)* |

**MEKpp phosphorylates ERK:**

| $k_{on}$ | $16.304 \times 10^6 M^{-1}s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |
| $k_{off}$ | $4.6 s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |
| $k_{cat}$ | $0.2 s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |

**MEKpp phosphorylates ERK$_T$ or ERK$_Y$:**

| $k_{on}$ | $16.304 \times 10^6 M^{-1}s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |
| $k_{off}$ | $0.6 s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |
| $k_{cat}$ | $0.15 s^{-1}$ | (Ferrell and Bhatt, 1997)$^{[16]}$ |

**MKP3 dephosphorylates ERK$_{TY}$:**

| $k_{on}$ | $15 \times 10^6 M^{-1}s^{-1}$ | (Zhao and Zhang, 2001)$^{[17]}$ |
Table 3: continued

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{off}}$</th>
<th>$k_{\text{cat}}$</th>
<th>(Zhao and Zhang, 2001)[17]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.235 s$^{-1}$</td>
<td>0.14 s$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

**MKP3 dephosphorylates ERK$_T$ on Thr183:**

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{on}}$</th>
<th>$k_{\text{off}}$</th>
<th>$k_{\text{cat}}$</th>
<th>(Zhao and Zhang, 2001)[17]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$15 \times 10^6$ M$^{-1}$s$^{-1}$</td>
<td>0.46 s$^{-1}$</td>
<td>0.099 s$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

**MKP3 dephosphorylates ERK$_Y$ on Tyr185:**

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{on}}$</th>
<th>$k_{\text{off}}$</th>
<th>$k_{\text{cat}}$</th>
<th>(Zhao and Zhang, 2001)[17]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$15 \times 10^6$ M$^{-1}$s$^{-1}$</td>
<td>0.25 s$^{-1}$</td>
<td>0.141 s$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

**STEP dephosphorylates ERK$_Y$ on Tyr185:**

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{on}}$</th>
<th>$k_{\text{off}}$</th>
<th>$k_{\text{cat}}$</th>
<th>(Zhou et al., 2002)[18]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$15 \times 10^6$ M$^{-1}$s$^{-1}$</td>
<td>7.926 s$^{-1}$</td>
<td>0.774 s$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

**STEP dephosphorylates ERK$_{TY}$ on Tyr185:**

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{on}}$</th>
<th>$k_{\text{off}}$</th>
<th>$k_{\text{cat}}$</th>
<th>(Zhou et al., 2002)[18]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$15 \times 10^6$ M$^{-1}$s$^{-1}$</td>
<td>5.88 s$^{-1}$</td>
<td>1.02 s$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: continued

**PP2A or PP2Ap dephosphorylates ERK on Thr183:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$8 \times 10^6 \text{M}^{-1}\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$149\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$27\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
</tbody>
</table>

**PP2A or PP2Ap dephosphorylates ERK on Thr183:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$8 \times 10^6 \text{M}^{-1}\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$175\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$1.22\text{s}^{-1}$</td>
<td>(Zhou et al., 2002)[18]</td>
</tr>
</tbody>
</table>

**Tautomycetin inhibits PP1:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$2.5 \times 10^7 \text{M}^{-1}\text{s}^{-1}$</td>
<td>estimated[19]</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$0.04\text{s}^{-1}$</td>
<td>estimated[19]</td>
</tr>
</tbody>
</table>

**Cyclosporin A inhibits PP2B:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$1.8 \times 10^7 \text{M}^{-1}\text{s}^{-1}$</td>
<td>estimated[20]</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$0.09\text{s}^{-1}$</td>
<td>estimated[20]</td>
</tr>
</tbody>
</table>

**Concentrations:**

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{CaM}]$</td>
<td>$3 \times 10^{-5}\text{M}$</td>
<td>(Kakiuchi et al., 1982)[21]</td>
</tr>
<tr>
<td>$[\text{Ca}^{2+}]_{basal}$</td>
<td>$8 \times 10^{-8}\text{M}$</td>
<td>(Allbritton et al., 1992)[22]</td>
</tr>
<tr>
<td>$[\text{CBP}_{fast}]$</td>
<td>$8 \times 10^{-5}\text{M}$</td>
<td>(Naoki et al., 2005)*</td>
</tr>
<tr>
<td>$[\text{CBP}_{medium}]$</td>
<td>$8 \times 10^{-5}\text{M}$</td>
<td>(Naoki et al., 2005)*</td>
</tr>
<tr>
<td>$[\text{CBP}_{slow}]$</td>
<td>$2 \times 10^{-5}\text{M}$</td>
<td>(Naoki et al., 2005)*</td>
</tr>
</tbody>
</table>
Table 3: continued

<table>
<thead>
<tr>
<th>Compartment</th>
<th>Concentration</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>[CBP&lt;sub&gt;sow&lt;/sub&gt;]</td>
<td>$2 \times 10^{-5} M$</td>
<td>(Naoki et al., 2005)*</td>
</tr>
<tr>
<td>[PP2B]</td>
<td>$1.6 \times 10^{-6} M$</td>
<td>(cf. Goto et al., 1986)&lt;sup&gt;[23]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[cAMP]</td>
<td>$2 \times 10^{-8} M$</td>
<td>(cf. Bacskaï et al., 1993)&lt;sup&gt;[24]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[R&lt;sub&gt;2&lt;/sub&gt;C&lt;sub&gt;2&lt;/sub&gt;]</td>
<td>$1 \times 10^{-6} M$</td>
<td>(cf. Hofmann et al., 1977)&lt;sup&gt;[25]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[PDE]</td>
<td>$0.5 \times 10^{-6} M$</td>
<td>(Bhalla and Iyengar, 1999)*</td>
</tr>
<tr>
<td>[CDK5]</td>
<td>$2 \times 10^{-7} M$</td>
<td>(Fernandez et al., 2006)*</td>
</tr>
<tr>
<td>[PP2A]</td>
<td>$2 \times 10^{-7} M$</td>
<td>(Fernandez et al., 2006)*</td>
</tr>
<tr>
<td>[PP1]</td>
<td>$2 \times 10^{-6} M$</td>
<td>(cf. Ingebritsen et al., 1983)&lt;sup&gt;[26]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[DARPP-32]</td>
<td>$5 \times 10^{-5} M$</td>
<td>(estimated from Halpain et al., 1990)&lt;sup&gt;[27]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[STEP&lt;sub&gt;p&lt;/sub&gt;]</td>
<td>$1 \times 10^{-6} M$</td>
<td>this study*</td>
</tr>
<tr>
<td>[Raf]</td>
<td>$1.8 \times 10^{-7} M$</td>
<td>(Bhalla and Iyengar, 1999)*</td>
</tr>
<tr>
<td>[MEK]</td>
<td>$1.2 \times 10^{-6} M$</td>
<td>(Huang and Ferrell, 1996)*&lt;sup&gt;[28]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[ERK]</td>
<td>$1.2 \times 10^{-6} M$</td>
<td>(Huang and Ferrell, 1996)*&lt;sup&gt;[28]&lt;/sup&gt;</td>
</tr>
<tr>
<td>[MKP3]</td>
<td>$0.0033 \times 10^{-6} M$</td>
<td>this study*</td>
</tr>
</tbody>
</table>

Compartment:

<table>
<thead>
<tr>
<th>Volume</th>
<th>Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>spine volume</td>
<td>$10^{-15} l$</td>
</tr>
</tbody>
</table>
Table 3: List of parameters used for simulation of the MAP kinase model that concerns PP1 or calcineurin (PP2B) activates STEP. Asterisk indicates that this reference refers to a computational model. [1]: Parameters were estimated according to Markram et al. (1998), and adjusted to maintain the resting calcium level at 10 nM. [2]: Markram et al. (1998) estimated these parameters according to Falke et al. (1994). [3]: D’Alcantara et al. (2003) estimated these parameters according to Linse et al. (1991). [4,8,9,10,11,12,13,14,16,17,18]: Parameters were obtained from in vitro experiments. [5]: Parameters were estimated according to basal cAMP concentration in sensory neurons of marine snail *Aplysia*, measured by Bacskai et al. (1993).[6]: Bhalla and Iyengar (1999) estimated these parameters according to experimental data obtained by Hasler et al. (1992). [7]: Bhalla and Iyengar (1999) estimated these parameters according to experimental data obtained by Ogreid and Døskeland (1981). [19,20]: Parameter was estimated according to IC50. [21]: Measurement was obtained from cerebral cortex tissue extract from rat.[22]: Data was obtained from cytosolic extract from Xenopus laevis oocytes. [23]: Measurement was obtained from dorsal striatum tissue extract from rat. [24]: Measurement was obtained from sensory neurons of marine snail *Aplysia*. [25]: Measurement was obtained from rabbit brain extract. [26]: Measurement was obtained from rabbit brain extract.[27]: Measurement was obtained from immunoblotting of striatal slices of rats. [28]: Huang and Ferrell (1996) estimated these concentrations according to concentrations of corresponding molecules in *Xenopus oocytes* by Matsuda et al. (1992). [29]: Measurement was based apical spine of CA1 pyramidal neuron.
### Table 4: List of abbreviations used in MAP kinase model that concerns PP1 or calcineurin (PP2B) activates STEP.

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Definition/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaM</td>
<td>calmodulin</td>
</tr>
<tr>
<td></td>
<td>UniProt:P62158</td>
</tr>
<tr>
<td>PP2B</td>
<td>calcineurin/protein phosphatase 2B</td>
</tr>
<tr>
<td></td>
<td>GO:0005955</td>
</tr>
<tr>
<td>cAMP</td>
<td>cyclic adenosine monophosphate</td>
</tr>
<tr>
<td></td>
<td>CHEBI:17489</td>
</tr>
<tr>
<td>R2C2</td>
<td>cyclic AMP-dependent protein kinase complex</td>
</tr>
<tr>
<td></td>
<td>GO:0005952</td>
</tr>
<tr>
<td>PDE</td>
<td>cyclic nucleotide phosphodiesterase</td>
</tr>
<tr>
<td></td>
<td>GO:0005966</td>
</tr>
<tr>
<td>CDK5</td>
<td>cyclin-dependent protein kinase 5</td>
</tr>
<tr>
<td></td>
<td>UniProt:Q03114</td>
</tr>
<tr>
<td>PP1</td>
<td>protein phosphatase 1</td>
</tr>
<tr>
<td></td>
<td>GO:0000164</td>
</tr>
<tr>
<td>PP2A</td>
<td>protein phosphatase 2A</td>
</tr>
<tr>
<td></td>
<td>InterPro:IPR06186</td>
</tr>
<tr>
<td>DARPP-32</td>
<td>dopamine- and cAMP-regulated phosphoprotein of 32 kDa</td>
</tr>
<tr>
<td></td>
<td>UniProt:Q9UD71</td>
</tr>
<tr>
<td>STEP</td>
<td>striatal-enriched tyrosine phosphatase</td>
</tr>
<tr>
<td></td>
<td>UniProt:P35234</td>
</tr>
<tr>
<td>Raf</td>
<td>RAF proto-oncogene serine/threonine-protein kinase</td>
</tr>
<tr>
<td></td>
<td>UniProt:P11345</td>
</tr>
<tr>
<td>MEK</td>
<td>MAPK/ERK kinase</td>
</tr>
<tr>
<td></td>
<td>UniProt Q01986</td>
</tr>
<tr>
<td>ERK</td>
<td>extracellular signal regulated kinases</td>
</tr>
<tr>
<td></td>
<td>InterPro:IPR008349</td>
</tr>
<tr>
<td>MKP</td>
<td>mitogen-activated protein kinase phosphatase</td>
</tr>
</tbody>
</table>
Table 4: List of abbreviations used in MAP kinase model that concerns PP1 or calcineurin (PP2B) activates STEP.

In both models DARPP-32 can be phosphorylated on Thr34 by cAMP-activated PKA, and on Thr75 by a basal level CDK5. Calcineurin, once activated by calcium and calmodulin, dephosphorylates DARPP-32 on Thr34. Thr75 of DARPP-32 can be dephosphorylated by PP2A. PP2A phosphatase activity increases upon its phosphorylation by PKA. PKA can also phosphorylate PDE, which in turn catalyses cAMP decomposition into adenosine monophosphate (AMP). Once DARPP-32 is phosphorylated on Thr34, it becomes a potent inhibitor of PP1 via a competitive inhibition mechanism; whereas when phosphorylated on Thr75, it becomes an inhibitor of PKA, also via a competitive inhibition mechanism.

A direct activation of Raf by calmodulin was used as a simplification to replace reactions describing calmodulin-activated Ras guanine nucleotide releasing factor protein (Ras-GRF), the subsequently activationg of Ras, then Raf. Dual phosphorylations of both MEK and ERK were modelled in a distributive manner, indicating that the kinases release their substrate after phosphorylation of the first residue, and are required to rebind to this substrate for phosphorylating the second residue (Ferrell and Bhatt, 1997). Two threonine sites of MEK are phosphorylated by activated Raf and dephosphorylated by PP2A. The phosphorylations of threonine and tyrosine residues of ERK are catalysed by dual-phosphorylated MEK and dephosphorylated by dual-specific phosphatase mitogen-activated protein kinase phosphatase (MKP), threonine/serine phosphatase PP2A and tyrosine phosphatase STEP. STEP is inactivated by PKA but activated by PP1 or calcineurin in the respective models.

For each computational model, several simulations were designed in order to compare simulation results with published experimental data for an animal model. The simulations included: 1) a train of high frequency calcium spikes, replicating the activation of NMDA
and AMPA receptors by glutamate input (Fig. 25, 100 spikes with a frequency of 50 Hz) (Sabatini et al., 2002); 2) a transient but prolonged cAMP spike, mimicking the effect of activated D1 receptor by dopamine. (Fig. 26) (Gonon, 1997); 3) the simultaneous inputs from both calcium spikes and cAMP, replicating the activation of both NMDA and dopamine D1 receptors (Fig. 27) (Valjent et al., 2005).

Figure 25: A train of calcium spikes. 100 calcium spikes are produced at 50 Hz.

**Simulation of PP1 activating STEP**

In the model where PP1 activates STEP, a train of calcium spikes led to the transient activation of ERK that reached maximum level after 5 minutes of stimulation and returned to basal activity within 1 hour, which is in agreement with published experimental data from Paul et al. (2003). Calcium also activated calcineurin and subsequent dephosphorylation of DARPP-32 on Thr34, resulting in reduced DARPP-32 inhibition of PP1 (Fig. 28). PP1 steadily activated STEP, and the activity of STEP peaked as much as 3 times more than its basal level, which roughly agrees with experimental measurement from Paul et al.
3.3 Phosphatase regulating STEP activity

Figure 26: A single spike of cAMP increase.

Figure 27: Stimulation by calcium and cAMP. 100 calcium spikes are introduced at 50 Hz, as well as a single spike of cAMP.
(2003). As a consequence, calcium-induced a transient ERK activation with the peak level only reaching about 15% of the total protein.

**Figure 28:** Effects of a train of Ca\(^{2+}\) spikes in the model where PP1 activates STEP. Time courses of various proteins after the simulation of 100 Ca\(^{2+}\) spikes at 50 Hz. ‘bar’ in the name of a protein indicates a value normalised by the total amount of that protein. Red: activated ERK. Green: phospho DARPP-32 on Thr\(^{34}\). Purple: activated calcineurin. Light blue: activated PP1. Yellow: activated STEP. Dark blue: intracellular Ca\(^{2+}\) concentration.

In this model, a transient cAMP pulse triggered DARPP-32 phosphorylation on Thr\(^{34}\) and further decreased PP1 phosphatase activity (Fig. 29). The consequence of this transient PP1 inhibition led to further reduction in STEP activity. However, since there was no upstream calcium input, there was no observable ERK activation.

The paired cAMP and calcium stimuli greatly enhanced ERK activity (Fig. 30). The maximal ERK activation reached approximately 50% of the total protein level, which was more than three-fold greater than the ERK activity induced by calcium alone. As demonstrated experimentally, increased ERK phosphorylation results from combined activation of NMDA and dopamine D\(_1\) receptor in the striatum (Valjent et al., 2005), which is in agreement with our paired in silico stimulation results. Enhanced ERK activity was due to the positive effect of cAMP on DARPP-32 Thr\(^{34}\) phosphorylation, which counter-
Figure 29: **Effects of a single cAMP spike in the model where PP1 activates STEP.** Time courses of various proteins after the simulation of a single cAMP spike. ‘bar’ in the name of a protein indicates a value normalised by the total amount of that protein. Dark blue: intracellular cAMP elevation; other colour codes are the same as in Fig. 28.
acted the dephosphorylation effect from calcineurin, resulting from calcium. Therefore, there was no additional PP1 inhibition release from DARPP-32 and no increase in STEP phosphatase ability.

Figure 30: Effects of a single cAMP pulse and a train of Ca\(^{2+}\) spikes in the model where PP1 activates STEP. Time courses of various proteins after the simulation of a single cAMP pulse and a train of Ca\(^{2+}\) spikes. ‘bar’ in the name of a protein indicates a value normalised by the total amount of that protein. Intracellular elevation of cAMP and Ca\(^{2+}\) are not represented here. Other colour codes are the same as in Fig. 28.

In contrast, paired stimuli failed to sufficiently trigger ERK phosphorylation when DARPP-32 was ‘knocked out’ from the model (Fig. 31). This result had also been observed experimentally, showing that ERK activation induced by both NMDA and D1 receptors is attenuated in DARPP-32 knock-out mice (Valjent et al., 2005).

In order to predict measurable outputs, PP1 or calcineurin was inhibited in silico, 50 minutes prior to calcium stimulation by adding 5 µM tautomycetin or 5 µM cyclosporin A correspondingly, which led to dramatically increased ERK activity in both situations when compared with calcium input alone (Fig. 32). Although, in this model, PP1 was the phosphatase activating STEP, inhibiting calcineurin led to the constant phosphorylation of DARPP-32 on Thr34, and therefore a very strong inhibition of PP1. This explains why increased ERK
Figure 31: Effects of a single cAMP pulse and a train of Ca^{2+} spikes in the model where PP1 activates STEP and DARPP-32 knock out. Time courses of various proteins after the simulation of a single cAMP pulse and a train of Ca^{2+} spikes. 'bar' in the name of a protein indicates a value normalised by the total number of that protein. Intracellular elevation of cAMP and Ca^{2+} are not represented here. Other colour codes are the same as in Fig. 28.
activity, resulting from calcineurin inhibition, cannot be used in support of the hypothesis that calcineurin phosphorylates STEP (Paul et al., 2003).

![Phosphatase regulating STEP activity](image)

Figure 32: **ERK activity after calcineurin or PP1 inhibition followed by a train of Ca^{2+} spikes, in the model where PP1 activates STEP.**

Protein phosphatase, either calcineurin or PP1, was inhibited after equilibrium and 50 minutes before Ca^{2+} input. ‘bar’ in the name of a protein indicates a value normalised by the total amount of ERK. Purple: ERK activation when calcineurin is inhibited. Red: ERK activation when PP1 is inhibited. Green: ERK activity with only Ca^{2+} input. Blue: intracellular Ca^{2+} concentration.

**Simulation of calcineurin activating STEP**

In the model where calcineurin activates STEP, a train of calcium spikes led to insufficient and transient activation of ERK. This is because activated calcineurin directly increased STEP activity, which in turn dephosphorylated ERK (Fig. 33).

The cAMP pulse alone was also insufficient to induce ERK activation (Fig. 34), whereas paired cAMP and calcium stimuli only slightly increased and prolonged ERK phosphorylation, when compared to a single calcium input (Fig. 35). Therefore, even the combined activa-
Figure 33: Effects of a train of Ca\textsuperscript{2+} spikes in the model where calcineurin activates STEP. Time courses of various proteins after the simulation of 100 Ca\textsuperscript{2+} spikes at 50 Hz. ‘bar’ in the name of a protein indicates a value normalised by the total amount of that protein. Red: activated ERK. Green: phospho DARPP-32 on Thr34. Purple: activated calcineurin. Light blue: activated PP1. Yellow: activated STEP. Dark blue: intracellular Ca\textsuperscript{2+} concentration.
tion of NMDA and dopamine D1 receptors was not sufficient to activate ERK. This is not in agreement with experimental data (Valjent et al., 2005).

![Figure 34: Effects of a single cAMP spike in the model where calcineurin activates STEP. Time courses of various proteins after the simulation of a single cAMP spike. 'bar' in the name of a protein indicates a value normalised by the total amount of that protein. Dark blue: intracellular cAMP elevation, other colour codes are the same as in Fig. 33.](image)

In the protein phosphatase inhibition experiments in silico, only inhibition of calcineurin significantly increased ERK activity, while PP1 inhibition had no effect (Fig. 36). Since DARPP-32 did not mediate this calcium induced STEP activation, inhibiting PP1 had no effect on STEP activity.

**Summary and conclusion**

Two computational models were constructed, to represent the different regulatory effects on STEP. By comparing the simulations with existing experimental measurements for each model, I have demonstrated how different model structures, whilst utilising the same parameter set, could produce different and distinct simulation results.
Figure 35: Effects of a single cAMP pulse and a train of Ca^{2+} spikes in the model where calcineurin activates STEP. Time courses of various proteins after the simulation of a single cAMP pulse and a train of Ca^{2+} spikes. 'bar' in the name of a protein indicates a value normalised by the total amount of that protein. Intracellular elevation of cAMP and Ca^{2+} are not represented here. Other colour codes are the same as in Fig. 33.
Figure 36: The ERK activity after calcineurin or PP1 inhibition followed by a train of Ca\(^{2+}\) spikes, in the model where calcineurin activates STEP. Protein phosphatase, either calcineurin or PP1, was inhibited after equilibrium and 50 minutes before Ca\(^{2+}\) input. ‘bar’ in the name indicates a value normalised by the total amount of ERK. Purple: ERK activation when calcineurin is inhibited. Red: ERK activation when PP1 is inhibited. Green: ERK activity with only Ca\(^{2+}\) input. Blue: intracellular Ca\(^{2+}\) concentration. Since the red line and green line overlapped, the red line is drawn thicker.
DARPP-32, once dephosphorylated by calcineurin, demonstrated reduced inhibition of PP1. The inhibition of calcineurin, as would be expected, increased DARPP-32 phosphorylation on Thr34, thereby increasing the inhibition of PP1. Therefore, experiments based on calcineurin inhibition are somewhat confusing, since the results reflect both the inhibition of calcineurin, and of PP1. This is illustrated in the simulations above, where calcineurin inhibition in both models resulted in increased ERK activity (Fig. 32,36).

In the model where PP1 activates STEP, ERK phosphorylation increased dramatically due to combined in silico stimulation from cAMP and calcium (Fig. 30). Furthermore, this activation was attenuated in simulations using a DARPP-32 knock-out (Fig. 31), which is in agreement with experimental observations (Valjent et al., 2005). In contrast, the model where calcineurin activates STEP could not reproduce these phenomena (Fig. 35). Therefore, it is possible that PP1, instead of calcineurin, activates STEP, exerting an inhibitory effect on ERK activity, and that DARPP-32 plays an important role in modulating this effect.

In order to further validate my model, I decided to apply PP1 inhibition on primary striatal neuron cultures, as shown in next section. The idea was to apply specific PP1 inhibitors before glutamate stimulation, then compare ERK activity with the situation when only glutamate is applied. If PP1 adopts inhibitory regulation on ERK via activating STEP, increased ERK phosphorylation upon PP1 inhibition should been observed.

3.4 Experimental investigation of PP1 inhibitors on ERK activation

ERK2 activity in 7-day neuronal cultures

The 7-day neuronal cultures were treated with glutamate (50 µM) for varying times and analysed by immunoblotting with the antibody recognising ERK1/2 when phosphorylated on both threonine and tyrosine residues. Basal ERK2 was mainly in the dephosphorylated form. Following glutamate stimulation, ERK2 phosphorylation reached its
Experimental investigation of PP1 inhibitors on ERK activation

Peak level within 2 to 5 minutes, and then decreased (Fig. 37 and Fig. 38).

The tautomycetin (TC) treatment followed by glutamate stimulation resulted in similar transient activation of ERK2 and there was no significant difference between treated and untreated sample at any time point (Fig. 37 and Fig. 38).

**Figure 37: ERK2 activity in response to PP1 inhibition and glutamate stimulation in 7-day neuronal cultures.** Activation of ERK2 after glutamate stimulation for the duration indicated on the x-axis. Green bars represent cultures only stimulated with glutamate. Red bars represent cultures incubated with tautomycetin (5 µM, 50 minutes) followed by glutamate stimulation. The data was collected from 7-day primary cultures of striatal neurons. Quantification of ERK2 activation was calculated using the ratio between phospho-ERK2 and corresponding total ERK2. Data are mean ± SEM (four repeats per group). Paired T-test indicated no significant difference between TC treated and control group, for all time points.

**ERK activity in 14-day neuronal cultures**

In contrast to 7-day cultures, 14-day primary cultures revealed a much higher basal ERK2 activity. Incubating neurons with glutamate (50 µM) for varying times resulted in a marked time-dependent decrease in
Experimental investigation of PP1 inhibitors on ERK activation

Figure 38: Western blot image of ERK1/2 activity in response to PP1 inhibition and glutamate stimulation in 7-day neuronal cultures. Western blot of samples extracted from 7-day culture, as described in 3.2. Control samples were compared with TC treated samples (5 μM, 50 minutes).

dual-phosphorylated ERK2, suggesting that ERK2 was rapidly inactivated following glutamate treatment (Fig. 39 and Fig. 40).

Surprisingly, PP1 inhibition by TC treatment in these neurons resulted in a much pronounced decrease in ERK2 activation, not only at basal condition but also at every glutamate treated time point compared with untreated group (Fig. 39 and Fig. 40). This result indicates that PP1 has a stimulating effect on ERK activity in 14-day cultures of striatal neurons.

TC can also inhibit PP2A nonspecifically (higher IC50 than for inhibiting PP1). To rule out such an effect, I examined the phosphorylation status of ERK2 after PP2A inhibition by incubating cultures with okadaic acid, followed by glutamate treatment. Immunoblotting analysis shows a strong increase in dual-phosphorylated ERK2 at basal time point (0 min), as well as at all glutamate-treated time points (Fig. 41). As PP2A, a major serine/threonine phosphatase, can directly dephosphorylate ERK2 on the threonine residue, the increase in ERK activity is not surprising. These data suggests that the TC treatment does not inhibit the effect of PP2A on ERK, and that results ob-
3.4 Experimental investigation of PP1 inhibitors on ERK activation

Figure 39: **ERK2 activity in response to PP1 inhibition and glutamate stimulation in 14-day cultured neurons.** Activation of ERK2 after glutamate stimulation for the duration indicated on the x-axis. Green bars represent the group only stimulated with glutamate. Red bars represent the group incubated with tautomycetin (5 µM, 50 min) followed by glutamate stimulation. The data was collected from 14-day primary striatal neuron cultures. Quantification of ERK2 activation was calculated using the ratio between phospho-ERK2 and total ERK2. Data are mean ± SEM (7 repeats per group), paired T test: **, *P*<0.01; ***, *P*<0.002
served are due to PP1 inhibition. Since, ERK2 activity still decreased over time, even after inhibition of PP2A, the glutamate-induced ERK2 dephosphorylation might be due to a glutamate-activated tyrosine phosphatase or to a glutamate-inhibited kinase, upstream of the MAP kinase pathway.

In order to confirm the results of the western blots, immunofluorescence experiments on 14-day cultured neurons were carried out. In the basal state, ERK2 was present primarily in the dual-phosphorylated form (Fig. 42). Incubation with glutamate for 10 minutes resulted in decreased ERK2 activity (Fig. 43). Neurons with TC treatment exhibited dramatically decreased ERK2 activity in the basal state (Fig. 44), and in 10-minute glutamate treated states (Fig. 45). The immunofluorescence results are summarised in Fig. 46.
Experimental investigation of PP1 inhibitors on ERK activation

Figure 41: **ERK2 activity in response to PP2A inhibition and glutamate stimulation in 14-day neuronal cultures.** Activation of ERK2 after glutamate stimulation for the duration indicated on the x-axis. Green bars represent the group only stimulated with glutamate. Red bars represent the group incubated with okadaic acid (500 nM, 50 min) followed by glutamate stimulation. The data was collected from 14-day primary striatal neuronal cultures. Quantification of ERK2 activation was calculated using the ratio between phospho-ERK2 and total ERK2. Data are mean ± SEM (3 repeats per group), paired T test: *, P<0.05
Figure 42: Immunofluorescence image of ERK1/2 activation at basal level in 14-day neuronal cultures. Immunofluorescence colabelling of MAPK2 (in green), activated ERK (in red), and nuclear (in blue).

Figure 43: Immunofluorescence image on ERK1/2 activation after 10 minutes glutamate stimulation in 14-day neuronal cultures. Immunofluorescence colabelling of MAPK2 (in green), activated ERK (in red), and nuclear (in blue).
Figure 44: Immunofluorescence image on ERK1/2 activation in response to PP1 inhibition in 14-day neuronal cultures. Immunofluorescence colabelling of MAPK2 (in green), activated ERK (in red), and nuclear (in blue). 14-day neuronal cultures were treated with tautomycetin for 50 minutes before fixation.

Figure 45: Immunofluorescence image on ERK1/2 activation in response to PP1 inhibition and glutamate stimulation in 14-day neuronal cultures. Immunofluorescence colabelling of MAPK2 (in green), activated ERK (in red), and nuclear (in blue). 14-day neuronal cultures were treated with tautomycetin for 50 minutes followed by glutamate for 10 minutes before fixation.
3.4 Experimental investigation of PP1 inhibitors on ERK activation

Edge Figure 46: **Summary of immunofluorescence experiments on ERK1/2 activation.** The cells were enumerated using immunofluorescence images following PP1 inhibition and glutamate stimulation, in 14-day neuronal cultures. Green bars represent the group stimulated only with glutamate. Red bars represent the group incubated with tautomycetin (5 µM, 50 min) prior to glutamate stimulation. Data are mean ± SEM (On average, 120 neurons were counted across 6 regions per coverslip. 3 repeats per group). paired T test: *, P=0.04. A more detailed description of the procedure is given in 3.2.
Summary and conclusion

Experiments, conducted on primary striatal neuron cultures, displayed a switch in glutamate influence on ERK2 activity, changing from activation to inhibition, after 7 and 14 days respectively. Surprisingly, in 14-day cultures, PP1 exerted a stimulatory effect on ERK2, instead of the predicted inhibitory effect by previous work (Valjent et al., 2005), and incorporated into my initial computational models. Furthermore, according to the immunofluorescence results above, the phosphorylation of ERK2 predominantly occurred in neuronal cells. Thus, some non-neuronal contribution cannot be the explanation for glutamate induced ERK2 dephosphorylation.

A possible explanation to these contradictory results might be the different neuronal differentiation states. This is evidenced, firstly, by the differential effects observed of glutamate on ERK2 activity, which varied with the culture duration. This may indicate a developing glutamate signalling pathway. Secondly, previous in vivo work with DARPP-32 knock-out mice demonstrated that PP1 indirectly exerts a negative effect on MAP kinase pathway (Valjent et al., 2005). The results obtained here were in vitro from primary cultured neurons, which may not be as mature as the neurons in a normal functioning brain. Would it be possible that, during neuronal differentiation, the effect of PP1 on ERK changes?

As highlighted previously (section 3.1), NR2A- and NR2B-containing NMDA receptors have opposite effects on ERK activity. The inhibitory effect exerted by NR2B-containing NMDA receptor is due to the activation, by CaMKII, of SynGAP, a synaptic-specific Ras-GAP protein. Since PP1 deactivates CaMKII, it prevents CaMKII’s negative regulation on the MAP kinase pathway. In short, 14-day cultured striatum neurons may not be mature, and may predominantly express NR2B-containing NMDA receptors. Therefore, in these neurons, PP1 would act as a positive regulator of ERK activation.
3.5 Dual influence of PP1 on ERK activity

Model structure and simulation plan

Based on my experimental results, as described above, and their possible explanation, a preliminary model was constructed (Fig. 47). In this model, the ratio of NR2A- and NR2B-containing NMDA receptors determines the amplitude and duration of calcium influx through NMDA receptors, because of their distinct gating and pharmacological properties (Cull-Candy et al., 2001; Naoki et al., 2005). NR2B-containing NMDA receptor also serves as a scaffold protein, coupling SynGAP and CaMKII, and thereby facilitating the phosphorylation of SynGAP by activated CaMKII. CaMKII is activated by binding calmodulin, then autophosphorylates on Thr286, increasing its affinity for calmodulin. The activation of the MAP kinase pathway is initiated by calmodulin activating Ras-GRF, which in turn catalyses RasGDP phosphorylation to generate RasGTP. RasGTP binds and activates Raf, which then activates MEK, thereby stimulating ERK. ERK can be dephosphorylated by dual-specific phosphatase MKP, threonine/serine phosphatase PP2A and tyrosine phosphatase STEP. In this model, PP1 displays two modulations of the MAP kinase pathway: on the one hand, PP1 stimulates STEP, deactivating ERK; on the other hand, PP1 dephosphorylates CaMKII, inhibiting SynGAP, thereby stimulating ERK. As this model is the expanded version of the MAP kinase model described before (3.3), only the additional quantitative parameters are listed in Table 5, and additional abbreviations are explained in Table 6. This model and the Python script for simulation in E-Cell are available online (http://www.ebi.ac.uk/~luli/thesis/mapk/dual/).

In the following section, I present results obtained from two different models, where either NR2B- or NR2A-containing NMDA receptor is predominantly expressed. All other parameters remain exactly the same. Several simulation experiments were designed to replicate the stimuli used in experiments on primary cultures. The in silico stimulations for each model include: 1) a train of low-frequency calcium pulses, applied as a background signal, to replicate basal spontaneous glutamate activity. This mimics a relatively high ERK basal activ-
ity, corresponding to that observed experimentally in 14-day cultured neurons; 2) a group of calcium spikes at high frequency, in order to model the effect of glutamate stimulation; 3) the PP1 inhibitor, tautomycetin, introduced 50 minutes before the high frequency calcium inputs, to duplicate protein phosphatase inhibition experiments.

By using a computational modelling approach, I tested to ascertain whether the type of NMDA receptors expressed could cause a switch in the roles of PP1 on ERK activity.
Figure 47: **Computational model of the dual effect of PP1 on ERK activity.** PP1 inhibitory and stimulatory effects on ERK1/2 are depicted in green and red respectively. For clarity, the DARPP-32 pathway and other phosphatases of ERK1/2, except STEP, are not included in this diagram. The graphical conventions are those of the Process Description language of Systems Biology Graphical Notation (SBGN) (Le Novère et al., 2009). Rectangular box with rounded corners: macromolecule, circular container: simple chemical, elliptical container: state variable, "empty set": a source or sink, rectangle with cut-corners: complex, square on the arc: process, filled disc on the arc: association, filled arrow head: production, empty arrow head: stimulation, empty circle: catalysis.
Table 5: List of parameters used for simulation of the expanded MAP kinase model that simulates the dual regulatory roles of PP1 on ERK1/2. Only additional parameters that differ from those in Table 3 on page 65 are listed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>SynGAP binding NR2B:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on}$</td>
<td>$6 \times 10^8 \ M^{-1}s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$1 \ s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>CaMCa4 binding CaMKII:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on_{CaMKII}}$</td>
<td>$3.2 \times 10^6 \ M^{-1}s^{-1}$</td>
<td>(Tzortzopoulos et al., 2004)[1]</td>
</tr>
<tr>
<td>$k_{off_{CaMKII}}$</td>
<td>$0.343 \ s^{-1}$</td>
<td>(Tzortzopoulos et al., 2004)[1]</td>
</tr>
<tr>
<td>$k_{on_{CaMKII_{p}}}$</td>
<td>$3.2 \times 10^6 \ M^{-1}s^{-1}$</td>
<td>(Tzortzopoulos et al., 2004)[1]</td>
</tr>
<tr>
<td>$k_{off_{CaMKII_{p}}}$</td>
<td>$0.001 \ s^{-1}$</td>
<td>(Meyer et al., 1992)[2]</td>
</tr>
<tr>
<td>CaMKII autophosphorylation on Thr286:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{Thr286_{p}}$</td>
<td>$6.3 \ s^{-1}$</td>
<td>(Lučić et al., 2008)[3]</td>
</tr>
<tr>
<td>CaMKIIp binding NR2B:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on}$</td>
<td>$2.76 \times 10^8 \ M^{-1}s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$1 \ s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>CaMKIIp_NR2B phosphorylates SynGAP_NR2B:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>$1.0 \ s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>SynGAPp_NR2B dephosphorylation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>$0.02 \ s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>PP1 dephosphorylates CaMKII:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{on_{CaMKII_{p}PP1}}$</td>
<td>$3.0 \times 10^6 \ M^{-1}s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$k_{off_{CaMKII_{p}PP1}}$</td>
<td>$0.5 \ s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$k_{cat_{CaMKII_{p}PP1}}$</td>
<td>$2.0 \ s^{-1}$</td>
<td>(Zhabotinsky, 2000)*</td>
</tr>
</tbody>
</table>
### 3.5 Dual influence of PP1 on ERK activity

Table 5: continued

**CaM Ca4 activates RasGEF:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$3 \times 10^7 M^{-1}s^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$0.6 s^{-1}$</td>
<td>this study*</td>
</tr>
</tbody>
</table>

**RasGDP activation by CaM Ca4_RasGEF:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{cat}$</td>
<td>$2 s^{-1}$</td>
<td>(estimate from Sasagawa et al., 2005)*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$</td>
<td>$2 \times 10^{-8} M$</td>
<td>(estimate from Sasagawa et al., 2005)*</td>
</tr>
</tbody>
</table>

**RasGTP dephosphorylation by SynGAP:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$5 \times 10^6 M^{-1}s^{-1}$</td>
<td>Sasagawa et al., 2005)*[4]</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$4 s^{-1}$</td>
<td>Sasagawa et al., 2005)*[4]</td>
</tr>
<tr>
<td>$k_{cat}$</td>
<td>$10 s^{-1}$</td>
<td>Sasagawa et al., 2005)*[4]</td>
</tr>
</tbody>
</table>

**RasGTP autodephosphorylation:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{max}$</td>
<td>$1 \times 10^{-8} Ms^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$K_m$</td>
<td>$1 \times 10^{-6} M$</td>
<td>this study*</td>
</tr>
</tbody>
</table>

**RasGTP activates Raf :**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{on}$</td>
<td>$6 \times 10^7 M^{-1}s^{-1}$</td>
<td>Sasagawa et al., 2005)*</td>
</tr>
<tr>
<td>$k_{off}$</td>
<td>$0.5 s^{-1}$</td>
<td>Sasagawa et al., 2005)*</td>
</tr>
</tbody>
</table>

**Raf inactivation:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{max}$</td>
<td>$1 \times 10^{-8} Ms^{-1}$</td>
<td>this study*</td>
</tr>
<tr>
<td>$K_m$</td>
<td>$1 \times 10^{-6} M$</td>
<td>this study*</td>
</tr>
</tbody>
</table>

**Ca$^{2+}$ input :**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$5 \times 10^{-3} - 20 \times 10^{-3} Ms^{-1}$</td>
<td>$-rI/2FV$</td>
</tr>
</tbody>
</table>

**Numbers:**
Table 5: continued

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>$9.6485 \times 10^4 \text{ Cmol}^{-1}$</td>
<td>Faraday’s constant (cf. Racca et al., 2000)(^5)</td>
</tr>
<tr>
<td>total NR2 subunits</td>
<td>40</td>
<td>(cf. Racca et al., 2000)(^5)</td>
</tr>
<tr>
<td>$I$</td>
<td>$5 - 20 \text{ pA}$</td>
<td>(Naoki et al., 2005)(^*), depending on NMDA receptor composition</td>
</tr>
<tr>
<td>$r$</td>
<td>0.2</td>
<td>(Naoki et al., 2005)(^*), the fraction of current I carried by $\text{Ca}^{2+}$.</td>
</tr>
</tbody>
</table>

Concentrations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{CaMKII}]$</td>
<td>$7 \times 10^{-5} \text{ M}$</td>
<td>(cf. Petersen et al., 2003)(^7)</td>
</tr>
<tr>
<td>$[\text{SynGAP}]$</td>
<td>$5 \times 10^{-6} \text{ M}$</td>
<td>(cf. Cheng et al., 2006)(^8)</td>
</tr>
<tr>
<td>$[\text{RasGEF}]$</td>
<td>$1.8 \times 10^{-7} \text{ M}$</td>
<td>this study(^*)</td>
</tr>
<tr>
<td>$[\text{RasGDP}]$</td>
<td>$1.8 \times 10^{-7} \text{ M}$</td>
<td>(Bhalla and Iyengar, 1999)(^*)</td>
</tr>
</tbody>
</table>

Table 5: List of parameters used for simulation of the expanded MAP kinase model that simulates the dual regulatory roles of PP1 on ERK1/2. Only additional parameters that differ from those in Table 3 on page 65 are listed. Asterisk indicates that the reference refers to a computational model. [1,2,3]: Parameters were obtained from in vitro experiments. [4]: Sasagawa et al. (2005) estimated these parameters according to Gideon et al. (1992). [5]: Data was obtained from synaptic immunogold labelling for NMDARs of rats pyramidal neuron. [6]: Naoki et al. (2005) estimated these parameters according to Nimchinsky et al. (2004); Piña-Crespo and Gibb (2002). [7]: Measurement was obtained from immunogold labelling on isolated PSD extract from rats forbrain. [8]: Data was obtained from PSD fractions prepared from rat forebrains and cerebellums.
Distinct ERK activity induced by different compositions of NMDA receptors

High frequency calcium inputs caused a decrease of ERK activity when NR2B-containing NMDA receptors were highly expressed (Fig. 48), akin to the glutamate induced ERK dephosphorylation in 14-day neuronal cultures (Fig. 39). This model further supports the hypothesis that reduced ERK phosphorylation may be due to the activation of CaMKII, and the subsequent activation of SynGAP (Fig. 48). In contrast, where NMDA receptors wereentially composed of NR2A subunits, glutamate stimulated ERK activity. This is the result of the insufficiency NR2B subunits to couple CaMKII with SynGAP (Fig. 49). The low ERK activation peak in this figure is due to the basal activity of STEP.

Distinct responses induced by different compositions of NMDA receptors after PP1 inhibition

When PP1 was inhibited in the model where NMDA receptors are predominantly composed of NR2B, PP1 inhibition resulted in a reduction of basal ERK activity. Upon high frequency calcium stimulation,
3.5 Dual influence of PP1 on ERK activity

Figure 48: Effects of Ca\(^{2+}\) stimulation in the model where NR2B subunits constitute 90% of the total NR2 subunits. Time courses of indicated proteins after simulation of high frequency Ca\(^{2+}\) input, in a low frequency Ca\(^{2+}\) spike background. In this model, NR2B-containing NMDA receptors were composed of 90% of the total NMDA receptors. ‘bar’ in the name of a protein indicates a value normalised by the total amount of this protein. Green, activated ERK; Purple, phosphorylated SynGAP; Light blue, activated CaMKII; Dark blue, intracellular Ca\(^{2+}\).
Figure 49: Effects of Ca\textsuperscript{2+} stimulation in the model where NR\textsubscript{2A} subsunits constitute 90\% of the total NR\textsubscript{2} subunits. Time courses of indicated proteins after simulation of high frequency Ca\textsuperscript{2+} input, in a low frequency Ca\textsuperscript{2+} spike background. In this model, NR\textsubscript{2A}-containing NMDA receptors were composed of 90\% of the total NMDA receptors. ‘bar’ in the name of a protein indicates a value normalised by the total amount of this protein. Colour codes are the same as in Fig. 48.
dual phosphorylated ERK displayed a sharp but transient rise, which then fell steeply at a rate that was faster than without PP1 inhibition (Fig. 50). Thus, this model reproduces ERK activity as observed in 14-day cultures, and suggests a mechanism whereby PP1 could play a role to enhance ERK activity. The inhibition of PP1 switched on the sustained activation of CaMKII, which was then translated into the persistent activation of SynGAP via NR2B coupling (Fig. 51), thereby resulting in the inhibition of the Ras-ERK pathway. On the other hand, when NMDA receptors were predominantly composed of NR2A, the inhibition of PP1 significantly increased ERK activity following a high frequency calcium stimulation, because of the decreased STEP phosphatase activity (Fig. 52). Therefore, in the latter model, PP1 plays an inhibitory role on ERK activity. Overall, these two models, by switching the composition of NR2 subunits of NMDA receptor, suggest two opposing roles for PP1 in the regulation of the Ras-ERK pathway.

Figure 50: ERK activity after Ca\(^{2+}\) stimulation in the model where NR2B subunits constitute 90\% of the total NR2 subunits. Comparison of ERK activities between PP1 inhibited (red line) and PP1 not inhibited (green line) situations after Ca\(^{2+}\) stimulation. In this model, NR2B-containing NMDA receptors were composed of 90\% of the total NMDA receptors. ‘bar’ in the name indicates a value normalised by the total amount of ERK. Dark blue line represents intracellular Ca\(^{2+}\) concentration.
3.5 Dual influence of PP1 on ERK activity

Figure 51: SynGAP activity after Ca\textsuperscript{2+} stimulation in the model where NR2B subunits constitute 90\% of the total NR2 subunits. Comparison of SynGAP activities between PP1 inhibited (red line) and PP1 not inhibited (green line) situations after Ca\textsuperscript{2+} stimulation. In this model, NR2B- containing NMDA receptors were composed of 90\% of the total NMDA receptors. 'bar' in the name indicates a value normalised by the total amount of SynGAP.
Figure 52: ERK activity after Ca^{2+} stimulation in the model where NR2A subunits constitute 90% of the total NR2 subunits. Comparison of ERK activities between PP1 inhibited (red line) and PP1 not inhibited (green line) situations after Ca^{2+} stimulation. In this model, NR2A- containing NMDA receptors were composed of 90% of the total NMDA receptors. ‘bar’ in the name indicates a value normalised by the total amount of ERK. Dark blue line represents intracellular Ca^{2+} concentration.
Summary and conclusion

A preliminary model, containing two opposing regulatory roles of PP1 on the Ras-ERK pathway, has been constructed. Simulation results with two different compositions of NMDA receptors in NR2 subunits indicate that the role of PP1 can be dynamically balanced by the type of NMDA receptor expressed. In immature neurons, high expression levels of NR2B subunits effectively couple SynGAP with CaMKII, providing a positive control on the Ras-ERK pathway mediated by PP1; whereas in mature neurons, high expression levels of NR2A subunits shift the role of PP1 to that of a negative regulator of ERK.

The time courses of calcium-induced ERK dephosphorylation (without the inhibition of PP1) obtained in the simulation do not exactly correspond to experimental results at 30 minutes (Fig. 48 and Fig. 39). Nevertheless, simulation and experiments roughly agree. Most importantly, the distinct ERK responses to glutamate stimulation and the even more dramatic difference in response to PP1 inhibition, between the two NMDA receptor compositions, provide a plausible explanation for the dual roles of PP1 on ERK activity (Fig. 50 and Fig. 52).

3.6 Discussion

The MAP kinase pathway mediates persistent structural changes in MSN (Wu et al., 2001b; Park et al., 2003), acting as an important crosstalk between cortico-striatal glutamate input and mesencephalic dopamine signal (Girault et al., 2007), thus underlying drug-induced neuroadaptation. The amplitude and duration of ERK activation are important for regulating its targets. ERK activation relies on one dual-specific kinase, MEK, but its inactivation can be achieved through three classes of phosphatases, suggesting that diverse signals could be integrated at the level of these phosphatases. Moreover, this regulatory integration through phosphatase may be cell-specific.

In MSN, the predominant striatal cell type, ERK activation results from the combined stimulation of glutamate NMDA receptors and dopamine D1 receptors (Valjent et al., 2000, 2005; Salzmann et al., 2003). This integration is mediated by DARPP-32, a potent PP1 inhib-
itor. Thus many studies have focused on the possible PP1 inhibitory effect on STEP, a tyrosine phosphatase deactivating ERK in the MSN.

The present study addresses the roles of PP1 on regulating glutamate NMDA receptor-dependent modulation of ERK signalling pathway in the striatum. By comparing the simulations of alternative models, where either calcineurin or PP1 directly stimulates STEP, with existing experimental data, I have demonstrated that it is possible that PP1 directly stimulates STEP, exerting an inhibitory effect on ERK activity in MSN. However, PP1 can also stimulate ERK via inhibiting Ras-GAP activity, when NR2B-containing NMDA receptors are predominantly expressed. Therefore, dopamine regulated PP1 activity may exhibit opposite effects on MAP kinase pathway according to the state of neuronal differentiation, and in particular the composition of NMDA receptors.

Brain function relies on the chemical communication between neurons via synapses. The molecular composition of synapses changes over time, resulting in distinct intracellular signalling cascades between mature and nascent synapses (Petralia et al., 2005). One important aspect is the opposite effects of NMDA receptor activation on AMPA receptors, and therefore on synaptic weights (reviewed in Hall and Ghosh, 2008). In mature synapses, the activation and insertion of AMPA receptors are stimulated by the activation of NMDA receptors; whereas, in developing synapses, NMDA receptors negatively regulate AMPA receptor function. ERK plays a key role in mediating this regulation. As Zhu et al. (2002) demonstrated, CaMKII mediated AMPA receptor delivery to the synapse is enabled by the activation of Ras-ERK pathway. In developing synapses, NMDA signalling, through NR2B-containing receptors, permits the activation of SynGAP by CaMKII, thus inhibiting Ras-ERK pathway, and negatively regulating AMPA receptor recruitment to the postsynaptic membrane. On the contrary, in mature synapses, NMDA receptor signalling, via NR2A-containing receptors, induces the activation of ERK and positively regulates AMPA receptor insertion.

The model presented here, dealing with the changes of composition of NMDA receptors, provides a reusable and extendable base that can be used to investigate the consequences of neuronal differentiation and their effect on synaptic plasticity. In addition, this mo-
del sheds light on possible interactions between signalling cascades, particularly the two opposing regulatory effects of the dopamine signalling pathway to glutamate-mediated ERK activity.

The experiments conducted here on primary striatal neuron cultures after 7-day or 14-day show a switch of glutamate influence on ERK, from activation to inhibition. If immaturity is the explanation for NMDA receptor mediated inhibition on ERK for the 14-day cultures, we can propose an explanation for the glutamate-induced ERK activation in 7-day cultures. Kim et al. (2005) found a transition of NR2B function during neuronal development. Initially NR2B is critical for ERK activation. However, at a certain developmental stage, NR2B starts to mediate the downregulation of ERK activity (Kim et al., 2005). Indeed, NR2B-containing NMDA receptor, by itself, is sufficient for NMDA-induced ERK activation. However, the role it plays on regulating ERK largely depends on the proteins it associates with, and this varies with the developmental stage. Since the experiments presented here were performed using primary neuronal cultures, it is possible that the level of spontaneous neuronal activity is artificially high after 14-day cultures, and the mechanisms required to downregulate glutamate receptors are evoked. NR2B mediated SynGAP activation might be initiated as a tuning procedure. Another possible explanation would be that the interplay between dopamine and glutamate systems can be dynamically adjusted: with chronically high glutamate activity, dopamine would have an inhibitory effect on glutamate-induced ERK activity, via the inhibition of PP1’s positive effect on ERK pathway (via CaMKII and SynGAP); with activity-dependent glutamate input, dopamine would have a stimulating effect on glutamate induced ERK through the inhibition of PP1’s inhibitory effect on mitogen-activated protein kinase (MAPK) pathway (through STEP).

The molecular interactions incorporated in this model are all obtained from published experimental evidence in MSN. However, in order to verify my model, further experiments are required. First of all, NR2B antagonists (such as ifenprodil or Ro25-6981) could be applied to 14-day neuronal cultures, which would allow me to judge whether NR2B plays an inhibitory role on ERK at this stage. Secondly, more mature neurons should be put in cultures. The results of glutam-
3.6 Discussion

ate stimulation with or without PP1 inhibition would permit validation of my hypothesis that the neuronal differentiation stage shifts the directionality of PP1’s effect on ERK. However, healthy synaptic connections do not only depend on the length of the culture, but also on the culture system used. It is possible that the coordination between glutamatergic input and neurotrophin factors from cortical cells may be required for the maturation of MSN (Ivkovic and Ehrlich, 1999; Morrison and Mason, 1998; Poo, 2001). Therefore, co-culture may need to be considered. Thirdly, spontaneous neuronal activity induced high activation of ERK should be inhibited, for instance using tetrodotoxin.

DARPP-32 mediated PP1 inhibition, and the subsequent regulation of ERK activity, are key parts of the molecular mechanisms underlying dopamine controlled neuroplasticity (Girault et al., 2007). This regulation is dynamically adjusted depending on the type of NMDA receptor and its associated proteins. Furthermore, dopamine can also exert its effect after DARPP-32 translocation in the nucleus. Drugs of abuse cause DARPP-32 phosphorylation on Thr34, as well as dephosphorylation on Ser97, which results in a strong inhibition of PP1 in the nucleus (Stipanovich et al., 2008). Some interesting questions to address in the future are: How do dopamine and glutamate signals regulate ERK activity and translocation into the nucleus, and how does this nuclear PP1 inhibition further influence ERK and its targets? However, before addressing these questions, a preliminary issue to be considered is whether or not the same population of ERK is active at the synaptic level and travels all the way to the nucleus, or are there different pools of ERK? If the trafficking of upstream molecules of MAP kinase pathway from the spine to the cell body is involved (Harvey et al., 2008) and plays an important role in regulating ERK nuclear translocation, then dopamine regulation upstream of the ERK pathway may be more important than the synaptic regulation of ERK activity itself. A computational model based on multiple compartments should be able to provide possible answers to these questions in the future.
4.1 Dopamine modulation on synaptic plasticity in nucleus accumbens

Evidence suggests that drugs of abuse exert their addictive effects through modulating synaptic plasticity in reward-related neuronal circuits (Hyman et al., 2006; Kauer and Malenka, 2007). Within these circuits, NAc plays an important role, since glutamate transmission in NAc is responsible for drug-seeking behaviour (Kalivas, 2004; Self et al., 2004).

The GABAergic MSN, the predominant neuron type in NAc, is innervated by glutamate projections from limbic and cortical regions and by the dopamine inputs from the ventral mesencephalon. Therefore, through MSN, dopamine can directly affect glutamate transmission via the activation of second messenger pathways.

In MSN, both NMDA receptor-dependent LTP (Pennartz et al., 1993; Mulder et al., 1997; Schramm et al., 2002; Li and Kauer, 2004), and NMDA receptor-dependent LTD (Thomas et al., 2001) have been observed. Dopamine plays a crucial role in regulating these two forms of cortical striatal plasticities (Centonze et al., 1999; Calabresi et al., 1992; Choi and Lovinger, 1997). Briefly, dopamine, through D1- and D2-like receptors, affects the intracellular cAMP level, modulating PKA activity. A major relevant substrate for PKA is the DARPP-32. This is a potent PP1 inhibitor once phosphorylated on threonine 34.

In this thesis, I have shown that the bidirectional alteration of short-term synaptic plasticity was regulated by the frequency of postsynaptic calcium inputs. I further proposed that dopamine, via the cAMP-PKA-DARPP-32 pathway, not only shifted CaMKII activation towards low-frequency calcium spikes, but also extended its activation level at high frequencies, thus potentiating glutamatergic synapses. This finding is supported by experiments showing that DARPP-32 inhibition of PP1 is required for inducing NMDA receptor-dependent LTP.
in MSN (Calabresi et al., 2000). In rat striatal slices, the induction of LTP requires co-occurrence of excitatory inputs with dopamine D1 receptor activation (Wickens et al., 1996; Kerr and Wickens, 2001). However, regulating the potency of PP1 is just one mechanism by which dopamine modulates corticostriatal synaptic plasticity.

A crucial property of MSN in NAc and the dorsal striatum is that the basal hyperpolarized membrane potential ("down" state) can be periodically driven to a more depolarized state ("up" state), by excitatory input from cerebral cortex and thalamus (Wilson and Kawaguchi, 1996). When the synaptic input is weak, dopamine, via the activation of D1 receptor, increases K+ channel current, suppressing excitability; whereas when MSN is in "up" state, dopamine, through D1 receptor, sustains this depolarized state by enhancing L-type Ca2+ channel currents (Nicola et al., 2000). This way, dopamine increases the signal-to-noise ratio and magnifies the strongest active input (Schultz, 2002). In addition, acute stimulation of D1 receptor increases GluR1-containing AMPA receptor externalization through a PKA-dependent mechanism (Wolf et al., 2004). These observations, together with my findings, lead to the hypothesis that the transient activation of D1 receptor, if happening in conjunction with excitatory synaptic transmission, can enhance the effect of glutamate input, facilitating LTP in MSN.

Contrary to the D1 receptor, the D2 receptor exerts the opposite effect on cAMP, and inhibits LTP (Calabresi et al., 1997). However, D1 and D2 receptors are both required for the induction of LTD in MSN, despite the fact that their activations are induced in different cellular subtypes (Calabresi et al., 2000). A proposed scheme is that dopamine stimulates nitric oxide (NO) production via D1 receptors in the striatal NO synthesis-positive interneurons (a minority neuron type in the striatum) (Morris et al., 1997). NO elevates intracellular cyclic guanosine monophosphate (cGMP) levels through soluble guanylyl cyclase (sGC), and cGMP in turn activates cyclic GMP-dependent protein kinase (PKG). This NO-cGMP-PKG pathway may cooperate with postsynaptic D2 receptors to induce LTD (Centonze et al., 2001; Calabresi et al., 1999). PKG phosphorylates DARPP-32 on Thr34, inhibiting PP1 (Hemmings et al., 1984b). Furthermore, this PKG induced PP1 inhibition is necessary for LTD induction (Calabresi et al., 2000). Why is DARPP-32 phosphorylation on Thr34 required for inducing both
4.2 Structural plasticity in NAc

LTP and LTD? What are the other roles played by PKA and PKG in striatal synaptic plasticity, besides their different catalytic efficiencies on phosphorylating DARPP-32? These questions may be addressed by an extension of the current model to include cGMP second messenger pathway, and to verify whether an inactivated cAMP pathway combined with the increased cGMP level could trigger distinct dynamic behaviour of CaMKII activation.

4.2 Drugs of abuse induced structural plasticity in nucleus accumbens

Learning can result in altered morphology structure of neurons and synapses (Moser et al., 1994; Stewart and Rusakov, 1995; Leuner et al., 2003), thereby affecting synaptic efficacy (Horner, 1993; Rusakov et al., 1996; Nimchinsky et al., 2002). Drugs of abuse can produce persistent changes in the brain, and share many cellular and molecular mechanisms with learning. Repeated exposure to amphetamine and cocaine increases spine density and dendritic branching on the MSN of the NAc (Robinson and Kolb, 1997, 1999a; Robinson et al., 2001; Heijtz et al., 2003; Kolb et al., 2003; Li et al., 2003; Norrholm et al., 2003). These changes are persistent and can last up to three months after withdrawal of the drug (Kolb et al., 2003; Robinson and Kolb, 1999a; Robinson et al., 2001). Particularly on the MSN of the NAc, drugs of abuse increase the number of spines with multiple heads (Robinson and Kolb, 1997, 1999a). Research suggests that these MSNs are primarily influenced by addictive drugs, because both dopamine and glutamate signals are received by these neurons, and both are important for persistent experience-dependent plasticity (Robinson and Kolb, 2004; Kalivas, 1995; Wolf, 1998; Hyman and Malenka, 2001; Lamprecht and LeDoux, 2004).

Sustained structural plasticity requires the activation of ERK1/2 (Wu et al., 2001b). Acute amphetamine treatment has been shown to elicit the activation of MAP kinase and neuritic growth (Park et al., 2003). ERK1/2 activity can also be induced by acute cocaine injection and underlies early behavioural responses (Valjent et al., 2000). One of the best studied transcription factors underlying learning and memory, CREB, is an indirect target of ERK1/2 in the nucleus, indic-
ating that ERK1/2 can transduce drug induced temporal alterations into regulation of gene expression and protein synthesis. This results in long-term changes of the dendritic structure.

In this thesis, I have shown how the activation of ERK1/2 could serve as a convergence point between cortico-striatal glutamate inputs and mesencephalic dopamine signals. In mature MSN, the convergence of dopamine induced cAMP and the glutamate triggered Ca²⁺ leads to increased and prolonged ERK1/2 activity, which can be translated into a series of nuclear events. Hence, when glutamate and dopamine transmission happen concurrently, dopamine enhances, through D1 activation, cortically evoked excitation in postsynaptic MSN. This is accomplished not only by enhancing synaptic strength, but also by inducing long lasting events such as ERK1/2 mediated dendritic remodelling.

Drugs of abuse exert their effects by amplifying the function of the dopamine reward system. For instance, dramatic increases in extracellular dopamine levels (Giros et al., 1996) lead to prolonged excitation in dopamine neurons (Jones et al., 2000; Ungless et al., 2001). This results in enhanced glutamate signalling pathway activity in MSN and sustained structural plasticity, thereby producing substantial behavioural changes (West et al., 1997).

However, different drugs exert different effects on the dendritic structure, in different regions. Amphetamine and cocaine influence dendritic structure mainly on the distal portion of dendritic tree of the MSN (Robinson and Kolb, 1999a). Amphetamine has little effect on basilar dendrites of the prefrontal cortex pyramidal cells, while cocaine alters dendritic structure on both apical and basilar dendrites (Robinson and Kolb, 1997). Morphine markedly decreases spine density and branching in the NAc, in contrast to other drugs of abuse (Robinson and Kolb, 1999b; Robinson et al., 2002). To date, little is known about how different patterns of synaptic reorganisation affect the function of a neural circuit, and how altered functions contribute to drug induced behavioural outcomes. This poses an important challenge not only for wet lab experiments, but also for computational modelling. Large scale quantitative models based on the whole dendritic tree, which take into account the distribution of spines, the detailed signalling network, and the communication between neigh-
bouring spines, would provide valuable insight into what really happens in the addicted brain. In fact, such a model is currently being developed by a co-worker, where my work is adapted into the molecular network within each spine on MSN.

Drug addictions are complex and involve adaptations in multiple neuronal circuits that develop with different time courses (Kauer and Malenka, 2007). The work presented here sheds light on some biochemical cascades involved in striatal neuroadaptation, and provides computational models that can be reused and extended to study other aspects of this neuroadaptation.

4.3 Experience-dependent adaptation

An interesting finding of this work is the developing regulatory role played by glutamate, on ERK1/2 activity, which changed from stimulation in 7-day to inhibition in 14-day cultured MSN, and finally to stimulation in mature MSN.

Using a computational approach, I propose that this glutamate-dependent dynamic regulation is due to a compositional change of the NMDA receptor, and in particular the signalling molecules assembled by the NR2B and NR2A subunits at different neuronal differentiation stages. In fact, either NR2A- or NR2B-containing NMDA receptors are able to induce ERK1/2 activation. However, as the neuron develops, NR2B is coupled to an inhibitor of Ras-ERK1/2 pathway (Kim et al., 2005). Therefore, glutamate controlled ERK1/2 activity is closely correlated with the relative expression of NR2A and NR2B subunits.

Not only do the components and their organisations change during neuron development (Petralia et al., 2005), the molecular mechanisms underlying some major postsynaptic events also differ. For example, the insertion of AMPA receptors into the postsynaptic membrane is regulated differently in mature and developing neurons. AMPA receptors can be recruited to nascent synapses without the activation of NMDA receptor (Colonnese et al., 2003; Groc et al., 2006). In addition, NMDA receptors then negatively regulate AMPA receptor recruitment, the inverse of the situation that occurs in mature neuron (reviewed in Hall and Ghosh, 2008).
It is not clear, though, what are the reasons behind these functional changes. It might be an experience-expectant development, by which "neurons incorporate environmental stimuli to its normal pattern of development" (cited from Andersen, 2003). In this sense, during a certain period of neuronal differentiation stage, the intense glutamate input may induce adaptation in glutamatergic synapses, by adjusting the components of NMDA receptor. Thus, glutamate activates AMPA receptors, which trigger the activation of NMDA receptors that in turn positively regulate the insertion of AMPA receptors to postsynaptic membrane, strengthening glutamate transmission.

Another example, albeit far remote from the topic of this thesis, relates visual experience and NMDA receptor development in kitten visual cortex. NMDA receptors are important for eye-specific geniculo-cortical axon segregation during visual cortex development in kittens (Fox et al., 1991). In a normal developing animal, this function is gradually attenuated as segregation concludes. However, when this segregation has been delayed by rearing the animal in darkness, the loss of NMDA receptor function is inhibited. This indicates that some critical steps in development are not programmed at the level of the genes, but caused via experience.

Adaptation occurs not only during development; LTP is a kind of adaptation, whereas dendritic structural changes due to long term drug exposure are of another kind. However, if a disruption happens during a critical development stage, it may have a greater impact than the same event occurring later in life. In mature MSN, dopamine, via inhibition of PP1, exerts a positive regulation on MAP kinase pathway. On the other hand, in developing MSN, dopamine, via the same PP1 inhibition, negatively modulates MAP kinase activity. There are emerging studies, based on human and animal models, suggesting that the effects of drugs of abuse on the developing brain are different from their effects on mature neuronal systems (Stanwood and Levitt, 2004). Developing brains are extremely plastic and vulnerable, thus prenatal cocaine exposure can lead to more permanent cellular responsiveness, and severe developmental delay (Andersen, 2003; Singer et al., 2002). Although drug induced adaptation in early life may result from a far more complicated mechanism, my research on opposing dopamine
effects on MAP kinase pathway can still give some insight into how dopamine switches its role during neuron differentiation.

4.4 Simulation approach and parameter estimation

As described in previous chapters, the main modelling approach used in this thesis is based on deterministic simulations of continuous ODE systems, and reactions are assumed to happen in a closed, homogeneous compartment whose volume remains constant. Whilst not without limitations, such an approach and assumption can provide a good approximation of the behaviour of a large scale complex system. However, there remain many challenges for the future development of this approach.

Parameter estimation and measurement

The minimum requirements for modelling biochemical reactions are the concentrations of species and the kinetic constants of reactions. However, only a small fraction of these parameters were directly measured in vivo, the majority being obtained from experiments on purified biochemical entities. Clearly, it is even harder to obtain concentrations and reaction rates from a subcellular region, for example the dendritic spine itself. In most situations, modellers need to estimate multiple parameters from the experimental data of a single time course. This brings challenges for modelling, for instance, how to integrate measurements from different experiments, and how to constrain the parameter space and obtain reasonable estimations.

One future objective would be to work on multidimensional parameter estimations for large scale kinetic models. There are already many optimisation algorithms that are available or have been implemented (reviewed in Mendes and Kell, 1998). Some of these are based on a local search around the given initial estimations, for example the Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1963) and the downhill simplex method (Nelder and Mead, 1965). Others apply a global algorithm that continually updates the sequence of the lower and upper bounds of estimated parameters (Esposito and Floudas, 2000), and some adapt the hybrid generic algorithm that first
identifies the initial guesses and then locally optimises the parameters (Wolf and Moros, 1997; Balland et al., 2000; Wang and Kim, 2001; Teh and Rangaiah, 2002).

Another exciting area with potential application of this problem is real-time imaging, based on fluorescence resonance energy transfer (FRET) (Tsien et al., 1993). A reporter based on FRET between fluorescent proteins can be observed in specific subcellular regions of an intact cell without buffering signalling molecules. Quantitative analysis of FRET images can reveal kinase activity in living cells (Zhang et al., 2001; Ting et al., 2001; Sato et al., 2002). Based on the principle of FRET, Violin et al. (2003) designed a genetically encoded fluorescent reporter that can reversibly respond to protein kinase C (PKC) activity, and they revealed a calcium controlled oscillatory phosphorylation of PKC to histamine stimulation in HeLa cells. A characteristic of spine signalling is the high spatial and temporal resolution. By using FRET, Harvey et al. (2008) showed, in a single dendritic spine of pyramidal neuron, Ras activation triggered by LTP and the spread of Ras activity to neighbouring spines. These data can be used by parameter searching algorithms discussed above in order to constrain parameter space and obtain reasonable parameters for computational models.

*Multi-compartment and stochastic simulations*

Major signalling proteins in a spine are assembled near the postsynaptic membrane, forming a PSD (Collins et al., 2006). PSD is a highly dynamic structure. For example, during the induction of LTP, Thr286 phosphorylated CaMKII travels into the PSD, binding to the NMDA receptor (Strack et al., 1997b; Shen and Meyer, 1999; Strack et al., 2000; Bayer et al., 2001). Insertion of new AMPA receptors in the postsynaptic membrane is another major step of LTP (Hayashi et al., 2000; Lu et al., 2001; Shi et al., 2001). In addition, many active proteins diffuse out of the PSD to deliver the information carried by neurotransmitters to other parts of the spine, to neighbouring spines, or to the nucleus. The spine is therefore not a homogeneous compartment, since the PSD is a specific domain with distinct protein distribution. Nevertheless, PSD is also open, allowing signalling molecules to diffuse within the rest of the spine.
Hence, in computational models, should the PSD be considered as a subcellular compartment? A problem with this approach concerns the estimation of flux rates for molecules trafficking in and out of the PSD. It is crucial, since the speeds of trafficking can significantly influence the occurrence of other signalling events. It is also difficult to assess the volume of the PSD. Besides, does the shape of PSD remain constant? Moreover, the PSD consists mainly of proteins, causing massive molecular crowding and non-brownian diffusions, which invalidates mass-action law approaches.

Another approach is to consider the formation of PSD as a sequence of molecules binding to key scaffold proteins, such as PSD95, Shank, AKAP79/150 (A-kinase-associated protein of molecular weight 70 and 150 kDa), and MUPP1 (Kim and Sheng, 2004; Scannevin and Huganir, 2000; Sheng and Sala, 2001; Krapivinsky et al., 2004). Still, this strategy requires many estimations about binding and dissociation rates that are usually not available from biochemical experiments. Furthermore, we still know little about the organisations of these scaffolds, and it seems that many more are waiting to be discovered (Kennedy et al., 2005).

As explained above, the major signal transduction events in spine happen within a spatially limited region, the PSD, where molecules are present in a relatively low copy number. Therefore, random fluctuations would affect interactions in this region (Tölle and Le Novère, 2006). Hybrid systems that combine stochastic simulation for PSD-specific reactions with deterministic simulation for other reactions that involve larger populations can be applied. This arrangement will help address several questions in the future: what is the effect of signal transduction noise in regulating neuronal network? Are there any strategies that neurons adopt in order to filter out this noise?

Varying volumes

There is a strong correlation between synaptic plasticity and spine morphology. LTP induced enlargement of the spine head and shortening of spine neck have been reported in many experiments (review in Yuste and Bonhoeffer (2001); Lamprecht and LeDoux (2004)). Changes in spine morphology can contribute to enhanced synaptic
transmission due to increased glutamate receptor surface expression (Takumi *et al.*, 1999; Matsuzaki *et al.*, 2001). However, an immediate challenge for computational models based on dendritic spine is the treatment of inconstant volumes for the compartments, which change as a function of time and activity. This alters the concentrations of molecules, and subsequently influences reaction rates. Moreover, these observations weaken the assumption that the spine is a closed compartment that allows signalling molecules, for instance calcium, to undergo fast concentration changes during LTP induction (Nimchinsky *et al.*, 2002). It might still be true transiently in some microdomains nearby the receptor, however the widened spine neck can facilitate calcium diffusion into the dendritic shaft (Volfovsky *et al.*, 1999; Majewska *et al.*, 2000), and to the neighbouring spines. It might be a daunting task to model spine morphology changes, however, it will help us to predict the functional importance behind those changes.

Although building biologically realistic models is challenging and demanding, the development of computational models provides an opportunity to combine different pieces of knowledge together and to reveal the mechanisms underlying neuronal function. Each trial to reach a realistic representation and the associated quantitative analysis is valuable. As Kennedy *et al.* (2005) argued: "Rather than attempting to design experiments to distinguish between mediation and modulation, it will be more useful to ask questions about the quantitative contribution of each pathway to a change in synaptic strength."
BIBLIOGRAPHY


A. P. Barnes, D. Solecki, and F. Polleux. New insights into the molecular mechanisms specifying neuronal polarity in vivo. *Curr Opin*


S. Ivkovic and M. E. Ehrlich. Expression of the striatal DARPP-32/ARPP-21 phenotype in GABAergic neurons requires neuro-


M. E. Morrison and C. A. Mason. Granule neuron regulation of purkinje cell development: striking a balance between neurotrophin and


R. M. Mulkey, S. Endo, S. Shenolikar, and R. C. Malenka. Involvement of a calcineurin/inhibitor-1 phosphatase cascade in hippocampal long-term depression. *Nature*, 369(6480):486–488, 1994. doi:10.1038/369486a0. (Cited on pages 4, 18, 20, and 30.)


Y. H. Park, L. Kantor, B. Guptaroy, M. Zhang, K. K. W. Wang, and M. E. Gnegy. Repeated amphetamine treatment induces neurite outgrowth and enhanced amphetamine-stimulated dopamine release in rat pheochromocytoma cells (PC12 cells) via a protein


B. A. Perrino, A. J. Wilson, P. Ellison, and L. H. Clapp. Substrate selectivity and sensitivity to inhibition by FK506 and cyclosporin A of calcineurin heterodimers composed of the alpha or beta catalytic


G. Y. Wu, K. Deisseroth, and R. W. Tsien. Activity-dependent CREB phosphorylation: convergence of a fast, sensitive calmodulin kinase pathway and a slow, less sensitive mitogen-activated protein kinase...


LIST OF ACRONYMS

AC ............ adenylyl cyclase
AMP ............ adenosine monophosphate
AMPA ............ $\alpha$-amino-3-hydroxy-5-methyl-4-isoxazolepropionic acid
BDNF ........ brain-derived neurotrophic factor
CaMKII ........ calcium/calmodulin-dependent protein kinase II
cAMP .......... cyclic adenosine monophosphate
CDK5 .......... cyclin-dependent protein kinase 5
cGMP .......... cyclic guanosine monophosphate
CK1 ............ casein kinase 1
CK2 ............ casein kinase 2
CREB .......... cyclic AMP-response-element-binding protein
DARPP-32 ..... dopamine- and cAMP-regulated phosphoprotein of 32 kDa
ERK ............ extracellular signal regulated kinase
ERK1/2 ........ extracellular signal regulated kinases-1/2
ERK2 ............ extracellular signal regulated kinase-2
FRET .......... fluorescence resonance energy transfer
G$\alpha_{olf}$ ........ olfactory isoform of GTP-binding protein $\alpha$ subunit
GPCRs .......... G-protein coupled receptors
I1 ............ inhibitor I
LTD .......... long-term depression
LTP ............ long-term potentiation
MAL ............ Mass Action Law
MAP ............ mitogen-activated protein
MAPK ............ mitogen-activated protein kinase
MEK ............ MAPK/ERK kinase
MKP ............ mitogen-activated protein kinase phosphatase
MSK ............ mitogen- and stress-activated kinases
MSN ............ medium-sized spiny neuron
MTs ............ microtubules
NAc .......... nucleus accumbens
NMDA ....... \textit{N}-methyl-\textit{d}-aspartate
NMDAR ...... \textit{N}-methyl-\textit{d}-aspartate receptor
NO .......... nitric oxide
ODE .......... ordinary differential equation
PDE .......... cyclic nucleotide phosphodiesterase
PDE .......... partial differential equations
PKA .......... cyclic AMP-dependent protein kinase
PKC .......... protein kinase C
PKG .......... cyclic GMP-dependent protein kinase
PP1 .......... protein phosphatase 1
PP2A ............ protein phosphatase 2A
PP2B ............ protein phosphatase 2B
PP2C ............ protein phosphatase 2C
PSD .......... postsynaptic density
Ras-GAP ...... Ras GTPase activating protein
Ras-GEF . . . . Ras guanine nucleotide exchange factor protein
Ras-GRF . . . . Ras guanine nucleotide releasing factor protein
RSK . . . . . ribosomal protein S6 kinases
sGC . . . . . soluble guanylyl cyclase
SPRCs . . . . . synapse-associated polyribosome complexes
STEP . . . . striatal-enriched tyrosine phosphatase
SynGAP . . . synaptic Ras GTPase activating protein
TC . . . . . . tautomycetin
VTA . . . . . ventral tegmental area


• L. Li, M. I. Stefan, N. Le Novère. Postsynaptic calcium affects the ratio of PP2B and CaMKII activation. submitted
• L. Li, M. S. Matamales, N. Le Novère, J.-A. Girault. NMDA receptor controls PP1 dual regulations on ERK2 activity. *in preparation*
Colophon

This thesis was typeset with \LaTeX on the basis of André Miede’s style classicthesis, which is available for \LaTeX via CTAN as “classicthesis”.

Final Version as of 9th August 2010 at 18:04.